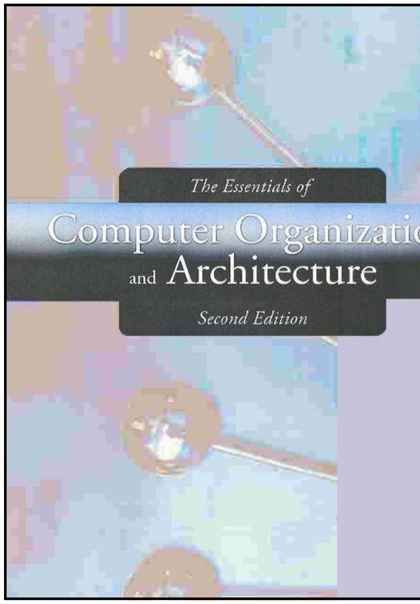

Chapter 3 Special Section

Focus on Karnaugh Maps

3A.1 Introduction

- To design a logic circuit, we start with the truth table. This is an example truth table; we needn't know what actual function it represents.
- Then we find the boolean function from the truth table:

$$F(x,y,z) = \bar{x}y\bar{z}+\bar{x}yz+x\bar{y}\bar{z}+xy\bar{z}+xyz$$

- All of the combos shown in this function represent the lines in the truth table which are = 1.
- Finally we simplify it using Boolean identities (laws to simplify boolean logic):

$$F(x,y,z) = x\bar{z}+y$$

x	y	z	F(x, y, z)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

3A.1 Introduction

- Simplification of Boolean functions leads to simpler (and usually faster) digital circuits.
- Simplifying Boolean functions using identities is time-consuming and error-prone.
- However, when using these identities, there are no rules on how or when to use the identities, and there is no well-defined set of steps to follow.
- This special section presents an easy, systematic method for reducing Boolean expressions.

3A.1 Introduction

- In 1953, Maurice Karnaugh was a telecommunications engineer at Bell Labs.
- While exploring the new field of digital logic and its application to the design of telephone circuits, he invented a graphical way of visualizing and then simplifying Boolean expressions.
- This graphical representation, now known as a **Karnaugh map**, or **Kmap**, is named in his honor.

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3A.2 Description of Kmaps and Terminology

- A Kmap is a matrix consisting of rows and columns of cells that represent the output values of a Boolean function.
- The output values placed in each cell are derived from the minterms of a Boolean function.
- A ***minterm*** is a product term that contains all of the function's variables exactly once, either complemented or not complemented.

3A.2 Description of Kmaps and Terminology

- For example, a function with 2 inputs *x* and *y* has 4 minterms: $\bar{X}\bar{Y}$, $\bar{X}Y$, $X\bar{Y}$, and XY corresponding to the 4 input combination values.
- Consider the Boolean function: $F(x,y) = XY + \bar{X}Y$
- Its minterms are:

- Only these two combos will have a value of 1 in the truth table.

Minterm	X	Y	F
$\bar{X}\bar{Y}$	0	0	0
$\bar{X}Y$	0	1	1
$X\bar{Y}$	1	0	0
XY	1	1	1

3A.2 Description of Kmaps and Terminology

- Similarly, a function having three inputs, has 8 minterms that are shown in this diagram.

Minterm	X	Y	Z
$\bar{X}\bar{Y}\bar{Z}$	0	0	0
$\bar{X}\bar{Y}Z$	0	0	1
$\bar{X}Y\bar{Z}$	0	1	0
$\bar{X}YZ$	0	1	1
$X\bar{Y}\bar{Z}$	1	0	0
$X\bar{Y}Z$	1	0	1
$XY\bar{Z}$	1	1	0
XYZ	1	1	1

3A.2 Description of Kmaps and Terminology

- A Kmap has a cell for each minterm.
- This means that it has a cell for each row for the truth table of a function.
- The truth table for the function *F(x,y) = xy* is shown at the right along with its corresponding Kmap.

$F(X,Y) = XY$

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X \ Y	0	1
0	0	0
1	0	1

3A.2 Description of Kmaps and Terminology

- As another example, we give the truth table and Kmap for the function, *F(x,y) = x + y* at the right.
- This function is equivalent to the OR of all of the minterms that have a value of 1. Thus

$$F(x,y) = X+Y = \bar{X}Y + X\bar{Y} + XY$$

The X+Y is the *simplified* form of the *standard* form shown to the right of the = sign.

Note: the 'not' **x** means x=0, the **x** means x=1.

F(X,Y) = X+Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

X \ Y	0	1
0	0	1
1	1	1

3A.3 Kmap Simplification for Two Variables

- Of course, the minterm function that we derived from our Kmap was not in simplest terms.
  - That's what we started with in this example.
- We can, however, reduce our complicated expression to its simplest terms by finding adjacent 1s in the Kmap that can be collected into groups that are powers of two.
- In our example, we have two such groups. (Can you find them ?)
- The best way of selecting two groups of 1s from our simple Kmap is shown in the next slide.

X \ Y	0	1
0	0	1
1	1	1

3A.3 Kmap Simplification for Two Variables

- There are two groups of 1s. Both groups are powers of two and they overlap.
- The next slide gives detailed guidance for selecting Kmap groups.
- Once the groups are found, we write down the simplied term for each group. A variable in a minterm can be removed if it changes value ***inside the group***:
- Red group: x

  Green group: y

  F(x,y) = x + y

X \ Y	0	1
0	0	1
1	1	1

3A.3 Kmap Simplification for Two Variables

Proof:

red group: xy'+ xy = x(y'+y) = x

green group: x'y + xy = (x'+x)y = y

F(x, y) = xy'+xy+x'y = xy'+xy+xy+x'y = x+y

X \ Y	0	1
0	0	1
1	1	1

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3A.3 Kmap Simplification for Two Variables

The rules of Kmap simplification are:

- Groupings can contain only 1s; no 0s.
- Groups can be formed only at right angles; diagonal groups are not allowed.
- The number of 1s in a group must be a power of 2, e.g., 2, 4, 8, etc., – even if it contains a single 1 (=2^0).
- The groups must be made as large as possible.
- Groups can overlap and wrap around the sides, top or bottom, of the Kmap.

3A.4 Kmap Simplification for Three Variables

- A Kmap for three variables is constructed as shown in the diagram below.
- We have placed each minterm in the cell that will hold its value.
  - Notice that the values for the *yz* combination at the top of the matrix form a pattern that is not a normal binary sequence. You can only change 1 digit from 1 item to the next; if you modify more than 1, the values can not be simplified.

X \ YZ	00	01	11	10
0	$\overline{X}\overline{Y}\overline{Z}$	$\overline{X}\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

3A.4 Kmap Simplification for Three Variables

- Thus, the first row of the Kmap contains all minterms where *x* has a value of zero.
- The first column contains all minterms where *y* and *z* both have a value of zero.

X \ YZ	00	01	11	10
0	$\overline{X}\overline{Y}\overline{Z}$	$\overline{X}\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

3A.4 Kmap Simplification for Three Variables

- Consider the function:

$$F(X,Y,Z) = \overline{X}\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$$

- If the function contains minterms only, each minterm corresponds to a 1 in the Kmap.
- Its Kmap is given below.
  - What is the largest group of 1s that is a power of 2? Answer on next slide.

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3A.4 Kmap Simplification for Three Variables

- This grouping tells us that changes in the variables *x* and *y* have no influence upon the value of the function: They are irrelevant.
- This means that the function,

$$F(X,Y,Z) = \overline{X}\,\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$$

reduces to *F(x) = z.*

You could verify this reduction with identities or a truth table.

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

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3A.4 Kmap Simplification for Three Variables

- Now for a more complicated Kmap. Consider the function:

$$F(X,Y,Z) = \overline{X}\,\overline{Y}\,\overline{Z} + \overline{X}\,\overline{Y}Z + \overline{X}YZ + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XY\overline{Z}$$

- Its Kmap is shown below. There are (only) two groupings of 1s.
  - Can you find them?

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

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3A.4 Kmap Simplification for Three Variables

- In this Kmap, we see an example of a group that wraps around the sides of a Kmap (envision the map as being drawn on a cylinder).
- This group tells us that the values of *x* and *y* are not relevant to the term of the function that is encompassed by the group.
  - What does this tell us about this term of the function?

What about the green group in the top row? See next slide.

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

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3A.4 Kmap Simplification for Three Variables

- The green group in the top row tells us that only the value of *x* is significant in that group (it is the only value which doesn't change ***within the group***).
- We see that it is complemented in that row, so the other term of the reduced function is $\overline{X}$.
- Full formula on next slide.
- Our reduced function is: $F(X,Y,Z) = \overline{X} + \overline{Z}$

Recall that we had six minterms in our original function!

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

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3A.4 Kmap Simplification for Three Variables

- Green group: x’y’z’ + x’y’z + xyz + x’yz’
  y & z both change, so they are dropped, leaving only x’.
- Red group: x’y’z’ + xy’z’ + x’yz’ + xyz’
  x & y change, so they are dropped, leaving only z’.
- Therefore, we are left with the simplified function

  f(x+y+z) = x’ + z’.

3A.5 Kmap Simplification for Four Variables

- Our model can be extended to accommodate the 16 minterms that are produced by a four-input function.
- This is the format for a 16-minterm Kmap.

WX \ YZ	00	01	11	10
00	$\overline{W}\overline{X}\overline{Y}\overline{Z}$	$\overline{W}\overline{X}\overline{Y}Z$	$\overline{W}\overline{X}YZ$	$\overline{W}\overline{X}Y\overline{Z}$
01	$\overline{W}X\overline{Y}\overline{Z}$	$\overline{W}X\overline{Y}Z$	$\overline{W}XYZ$	$\overline{W}XY\overline{Z}$
11	$WX\overline{Y}\overline{Z}$	$WX\overline{Y}Z$	$WXYZ$	$WXY\overline{Z}$
10	$W\overline{X}\overline{Y}\overline{Z}$	$W\overline{X}\overline{Y}Z$	$W\overline{X}YZ$	$W\overline{X}Y\overline{Z}$

3A.5 Kmap Simplification for Four Variables

- We have populated the Kmap shown below with the nonzero minterms from the function (again, we don’t need to know the purpose of this function.) The seven 1’s represent the seven minterms in the function):

$$F(W,X,Y,Z) = \overline{W}\overline{X}\overline{Y}\overline{Z} + \overline{W}\overline{X}\overline{Y}Z + \overline{W}\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + W\overline{X}\overline{Y}\overline{Z} + W\overline{X}\overline{Y}Z + W\overline{X}Y\overline{Z}$$

  – Can you identify (only) three groups in this Kmap?

Recall that groups can overlap.

WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

3A.5 Kmap Simplification for Four Variables

- Our three groups consist of:
  – A purple group entirely within the Kmap at the right.
  – A pink group that wraps the top and bottom.
  – A green group that spans the corners.
- Thus we have three terms in our final function:

$$F(W,X,Y,Z) = \overline{X}\overline{Y} + \overline{X}\overline{Z} + \overline{W}Y\overline{Z}$$

WX \ YZ	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

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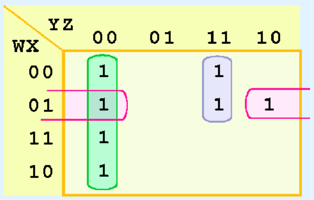
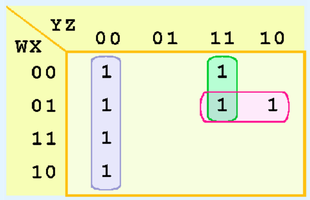
3A.5 Kmap Simplification for Four Variables

- It is possible to have a choice as to how to pick groups within a Kmap, while keeping the groups as large as possible.
- The (different) functions that result from the groupings below are logically equivalent.

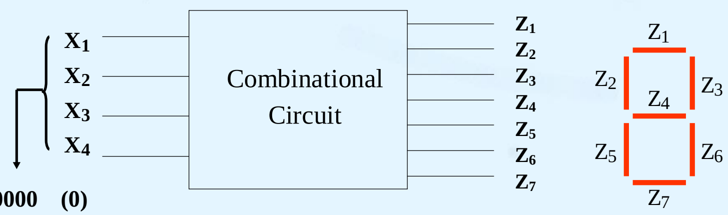




3A.6 Don't Care Conditions

- Real circuits don't always need to have an output defined for every possible input.
  - See the next slide for a 7-segment LED example. (Only 10 out of the 16 input combinations are defined.)
- If a circuit is designed so that a particular set of inputs can never happen, we call this set of inputs a *don't care* condition.
- They are very helpful to us in Kmap circuit simplification.

3A.6 Don't Care Conditions

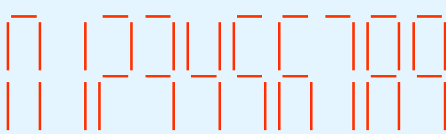


0000 (0)
0001 (1)
0010 (2)
0011 (3)
0100 (4)
0101 (5)
0110 (6)
0111 (7)
1000 (8)
1001 (9)

Don't Care Conditions:

1010 (X)
1011 (X)
1100 (X)
1101 (X)
1110 (X)
1111 (X)

Ten Output Patterns:



3A.6 Don't Care Conditions

- In a Kmap, a don't care condition is identified by an *X* in the cell of the minterm(s) for the don't care inputs, as shown below.
- In performing the simplification, we are free to include or ignore the *X*'s when creating our groups.

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don't Care Conditions

- In one grouping in the Kmap below, we have the function:

$$F(W,X,Y,Z) = \bar{W}\bar{X} + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don't Care Conditions

- A different grouping gives us the function:

$$F(W,X,Y,Z) = \bar{W}Z + YZ$$

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don't Care Conditions

- The truth table of:

$$F(W,X,Y,Z) = \bar{W}\bar{X} + YZ$$

is different from the truth table of:

$$F(W,X,Y,Z) = \bar{W}Z + YZ$$

- However, the values for which they differ, are the inputs for which we have don't care conditions.

$F(W,X,Y,Z) = \bar{W}\bar{X} + YZ$ grouping:

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

$F(W,X,Y,Z) = \bar{W}Z + YZ$ grouping:

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A Conclusion

- Kmaps provide an easy graphical method of simplifying Boolean expressions.
- A Kmap is a matrix consisting of the outputs of the minterms of a Boolean function.
- In this section, we have discussed 2- 3- and 4-input Kmaps. This method can be extended to any number of inputs through the use of multiple tables.

3A Conclusion

Recapping the rules of Kmap simplification:

- Groupings can contain only 1s; no 0s.
- Groups can be formed only at right angles; diagonal groups are not allowed.
- The number of 1s in a group must be a power of 2 – even if it contains a single 1 (=2^0).
- The groups must be made as large as possible.
- Groups can overlap and wrap around the sides of the Kmap.
- Use don't care conditions when you can.

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