

Chapters 2 Motion in One Dimension

Mechanics: Kinematics and Dynamics.

Kinematics deals with motion, but is not concerned with the cause of motion.

Dynamics deals with the relationship between force and motion.

Displacement

The word “displacement” implies the existence of an initial position (location) and a final position ... often of the same object but at different times.

The displacement, Δx , (as a result of an object’s motion in a period of time) is the vector that points from the initial position toward the final position, within a suitable frame of reference. It is the “change in position”.

The displacement of an object is not the same as the distance it travels.



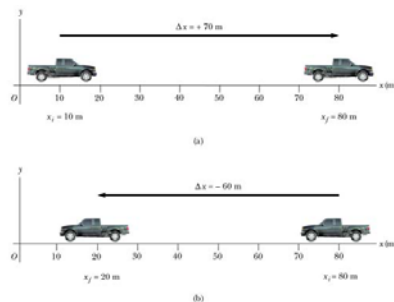
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Vector and Scalar

A **vector** is characterized by a magnitude and a direction. (velocity, displacement, ...)

A **scalar** quantity has only a magnitude but no direction. (mass, temperature, ...)

If the motion of an object is limited along a straight line (its motion is **one-dimensional**), the displacement of this object can have only two directions (one being opposite to the other). If a coordinate system is chosen for the displacement (as in defining which direction is positive), the displacement may appear to be scalar-like. However, one should note that the “sign” of the displacement contains information on “direction”.



$$\Delta x = x_f - x_i$$

Final position
(coordinate)

Initial position
(coordinate)

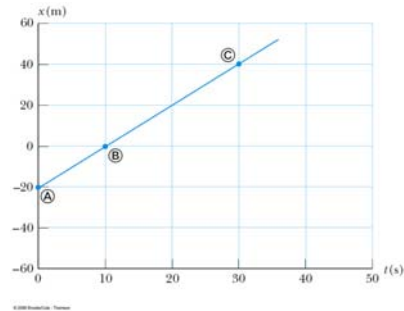
Average Velocity and Speed

average speed = distance traveled / elapsed time (scalar)

average velocity = displacement / elapsed time (vector)

Velocity and speed refer to the rate of change of an object's position (with respect to some stationary reference point, e.g. the origin).

If an object's average velocity (/speed) is zero for a period of time, does its average speed (/velocity) have to be zero for that period of time? **No (/Yes)**

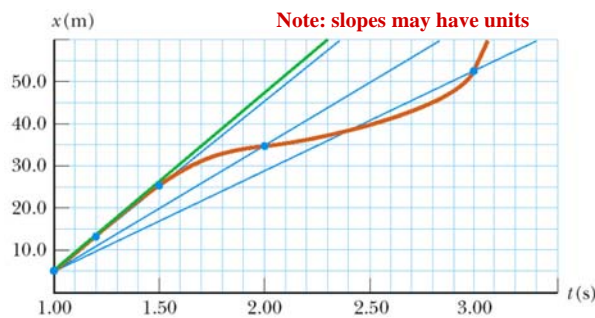


Avg. Velocity (1D) \leftrightarrow Slope Of Line Through Two End Points

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta(y \text{ value})}{\Delta(x \text{ value})}$$

average velocity

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$



In the figure, what is the average velocity between 1.00 s and 3.00 s? $\frac{52.5\text{ m} - 5\text{ m}}{3\text{ s} - 1\text{ s}} = 23.8\text{ m/s}$

However, average speed \neq | average velocity |

Instantaneous Velocity (1D): Slope Of Tangent

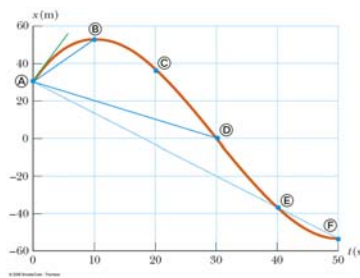
average velocity $\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$

instantaneous velocity

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

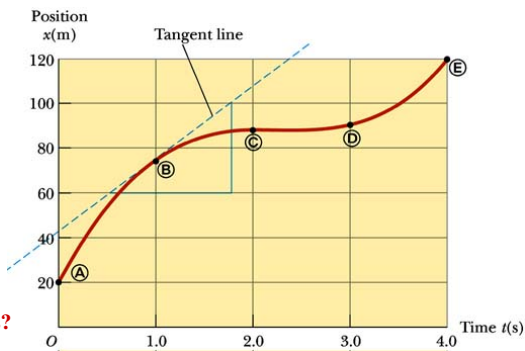
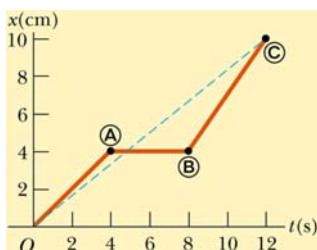
= slope of the tangent line to the x(t) plot

instantaneous speed $= |v|$



A straight line in the x(t) plot indicates constant velocity.

Instantaneous Velocity & Slope: Examples



What is the (instantaneous) velocity at t=2 s?

What is the instantaneous velocity at 1.0 s? $(100-60) \text{ m} / (1.8 \text{ s} - 0.5 \text{ s}) = 31 \text{ m/s}$

At which time is the instantaneous velocity the greatest: A, B, C or D?

Answer: A

What is the instantaneous velocity at 3.0 s? $\sim 12 \text{ m/s}$

Acceleration: the rate with which the velocity changes

Velocity is the rate of change of displacement (a vector).

Acceleration is the rate of change of velocity (also a vector).

The analysis of these two quantities is very similar.

The “rate of change” of a quantity is the “slope” of a plot of this quantity against time.

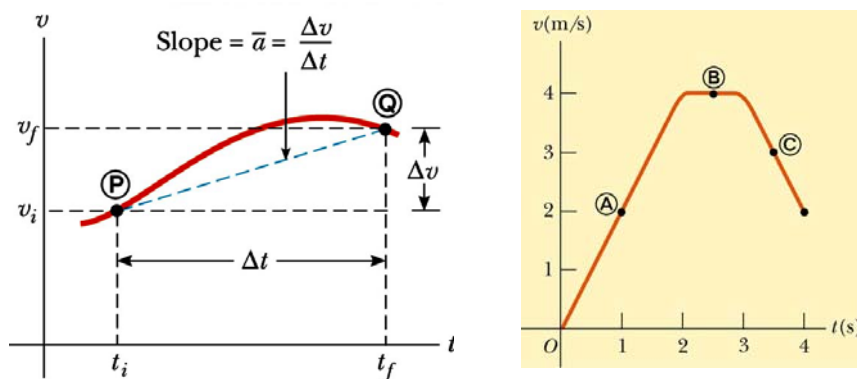
average acceleration
(vector)

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

instantaneous acceleration
(vector)

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

Acceleration Examples



A straight line in the $v(t)$ plot indicates constant acceleration.

A “negative acceleration” does not necessarily imply that the “speed” of an object is reducing.

Equations For Constant 1-D Acceleration

“Constant acceleration” means that a is constant (fixed).

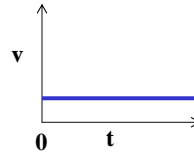
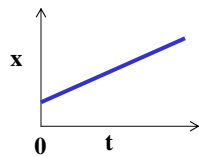
First, a trivial case ...

If the acceleration is zero, the velocity does not change with time, and the object will move with constant velocity, v .

If an object is at x_0 at time $t=0$, it will be found at position

$$x = x_0 + vt, \quad \Delta x = x - x_0 = vt$$

at time $t = t$.



Equations For Constant 1-D Acceleration

Non-zero constant acceleration: a (= constant)

Initial velocity at $t = 0$: v_0

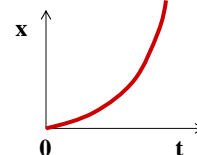
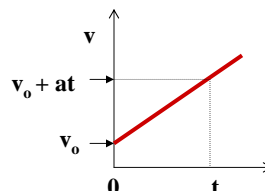
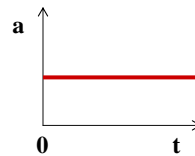
Velocity at time t is $v = v_0 + at$

The average velocity between time zero and time t is

$$\bar{v} = \frac{1}{2}(v_0 + v) = v_0 + \frac{1}{2}at$$

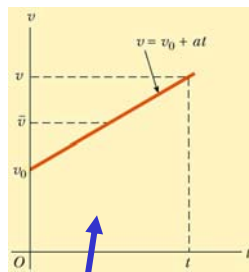
The total displacement the object between time 0 and time t is the product of the average velocity and the time elapsed (t). If the object is assumed to be at the origin at $t=0$, its position at time t is

$$\Delta x = \bar{v}t = v_0t + \frac{1}{2}at^2$$



More on Displacement in 1D

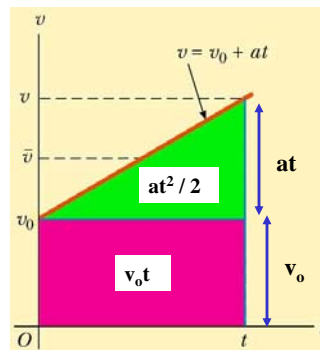
$$\Delta x = \bar{v}t = v_0 t + \frac{1}{2}at^2$$



area of a trapezoid is
 $(1/2) * (\text{top} + \text{base}) * \text{height}$

$$= (1/2) * (v_0 + v_0 + at) * t$$

$$= v_0 t + at^2 / 2$$



$$\text{area} = \Delta x = v_0 t + at^2 / 2$$

Displacement is the area under the v(t) curve.

Equations For Constant 1-D Acceleration

$$v = v_0 + at$$

No x

$$\Delta x = \frac{1}{2}(v_0 + v)t$$

No a

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

No v

Add one more equation \rightarrow

$$v^2 = v_0^2 + 2a\Delta x$$

No t

To get the position of the object at time t, we need to add the displacement to the initial position of the object

$$x = x_0 + \Delta x$$

Which Equation to Use?

A car is accelerating with an initial velocity of 1 m/s and a constant acceleration of 5 m/s². What is its velocity when it travels 50 m?

In 3 seconds, an object has accelerated from 0 to 20m/s with a constant acceleration. How much distance has it traveled during this time.

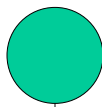
$$v = v_o + at$$

$$\Delta x = \frac{1}{2} (v_o + v) t$$

$$\Delta x = v_o t + \frac{1}{2} at^2$$

$$v^2 = v_o^2 + 2a \Delta x$$

Vertical Motion with Constant Gravitational Accel.



Acceleration due to gravity

$$g = 9.8\text{m/s}^2 \text{ downward}$$

Or, $a = -g = -9.8\text{m/s}^2$ (because "up" is usually defined as the positive y direction.)

This acceleration is seen for all objects on earth, big or small, heavy or light, in the absence of air resistance.

How to work on problems such as "how high does it go?" or

Velocity vanishes at the highest point.

"how long is it in the air?"

Displacement as specified in the question is reached!

$$v = v_o + (-g)t$$

$$\Delta y = \frac{1}{2} (v_o + v) t$$

$$\Delta y = v_o t + \frac{1}{2} (-g)t^2$$

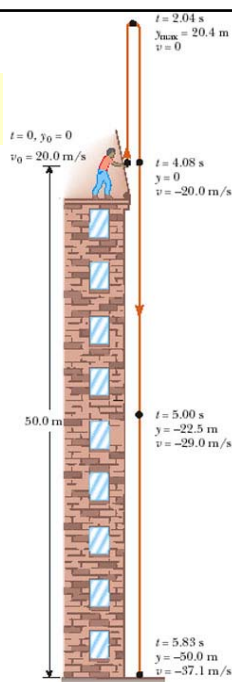
$$v^2 = v_o^2 + 2(-g) \Delta y$$

Equation $At^2 + Bt + C = 0$ has solutions: $t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$	Can usually tell which sign (+ or -) to use.
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Example Problems

1. A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. Determine (a) the time needed for the stone to reach its maximum height, (b) the maximum height, (c) the time needed for the stone to reach the ground.

2. In the Daytona 500 auto race, a Ford Thunderbird and a Mercedes Benz are moving side by side down a straightaway at 71.5 m/s. The driver of the Thunderbird realizes that she must make a pit stop, and she smoothly slows to a stop over a distance of 250 m. She spends 5.00 s in the pit and then accelerates out, reaching her previous speed of 71.5 m/s after a distance of 350 m. At this point, how far has the Thunderbird fallen behind the Mercedes Benz, which has continued at a constant speed?



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Review of Chapter 2

Displacement involves direction and distance (vector).

Average velocity = displacement/elapsed time

Instantaneous velocity = (average) velocity for an infinitesimally small time period.

Average acceleration = velocity variation / elapsed time

Instantaneous acceleration = velocity variation / infinitesimally small elapsed time

Equations for object undergoing constant acceleration in 1-D.

Free falling involves a constant acceleration of 9.8m/s^2 downward for all objects.