Chapter 6: Momentum And Collisions

The linear momentum p of an object is the product of the object's mass m and velocity v

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

SI Unit of Momentum: kg m / s or N s

When a single, constant force acts on the object, there is an **impulse** delivered to the object

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t$$

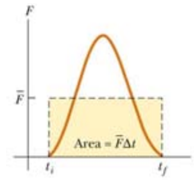
$\vec{\mathbf{I}}$ is defined as the *impulse.*

It's a vector quantity, the direction is the same as the direction of the force

Average force in impulse

The impulse of a force is the product of the average force $\overline{F}$ and the time interval Δt during which the force acts:

impulse = $\overline{F}$ Δt



Impulse-Momentum Theorem

F Δt = (m a) Δt = m (a Δt) = m Δv = Δ(mv)

IMPULSE – MOMENTUM THEOREM

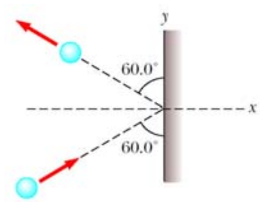
When a net force acts on an object, the impulse of the net force is equal to the change in momentum of the object.

Impluse = Change in momentum.

$$\overline{F}\,\Delta t = m\vec{v}_f - m\vec{v}_i$$

Example

A 3.00-kg steel ball strikes a massive wall at 10.0 m/s at an angle of 60.0° with the plane of the wall. It bounces off the wall with the same speed and angle. If the ball is in contact with the wall for 0.200 s, what is the average force exerted by the wall on the ball?



Impulse Applied to Auto Collisions

- The most important factor is the collision time, the time it takes the person to come to a rest
  - Smaller momentum change and longer impact time reduce the chance of dying in a car crash
- Ways to increase the time
  - Air bags

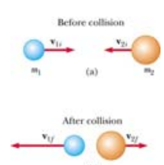
$$\overline{F}\,\Delta t = m\vec{v}_f - m\vec{v}_i$$

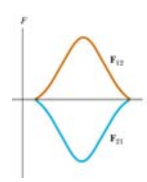
Conservation of Momentum

The total momentum of an isolated system is conserved.

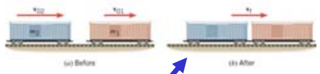
$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

- A result of Newton's Third Law.
- An isolated system is a system where the sum of all external forces is zero.



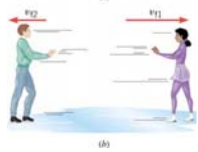


Conserv. Of Momentum (not necessarily KE)



total KE decreases

total KE increases

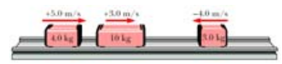


Momentum is conserved in either case.

Conservation of Momentum

- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Remember conservation of momentum applies to the *system*
- You must define the isolated system

Example: Three carts of masses 4.0 kg, 10 kg, and 3.0 kg move on a frictionless horizontal track with speeds of 5.0 m/s, 3.0 m/s, and 4.0 m/s. The carts stick together after colliding. Find the final velocity of the three carts.



Elastic and Inelastic Collisions

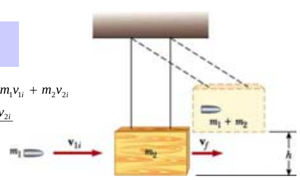
Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision

Inelastic collision -- One in which the total kinetic energy of the system is not the same before and after the collision.

Perfect 1D Inelas. Coll.

$$m_1v_f + m_2v_f = m_1v_{1i} + m_2v_{2i}$$

$$v_f = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2}$$



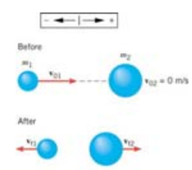
Collision in One Dimension (Elastic)

For any collision, momentum is conserved. We have

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + 0\cdot$$

For elastic coll., KE is conserved.

$$\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}m_1v_{1i}^2 + 0\cdot$$



Solving for v_{1f} and v_{2f}, we get

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i}$$

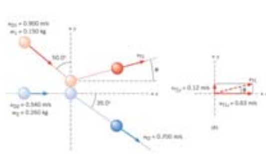
Perfect 1D Elastic Coll.

Collisions In Two Dimensions

Momentum is always conserved.

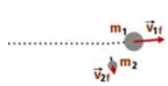
$$m_1v_{1fx} + m_2v_{2fx} = m_1v_{1ix} + m_2v_{2ix}\cdot$$

$$m_1v_{1fy} + m_2v_{2fy} = m_1v_{1iy} + m_2v_{2iy}$$



If kinetic energy is also conserved (i.e. elastic collision),

$$\frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2\cdot$$



Solve one dimensional elastic collison

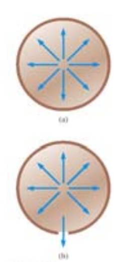
Rocket Propulsion

Cars, boats, airplanes accelerate by pushing against something (external). Rocket in space operates by discharging part of itself at high speed.

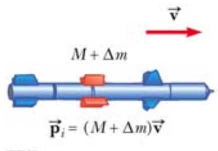
The rocket is accelerated as a result of the thrust of the exhaust gases.

This represents the inverse of an inelastic collision

- **Momentum is conserved**
- **Kinetic Energy is increased (at the expense of the stored energy of the rocket fuel)**

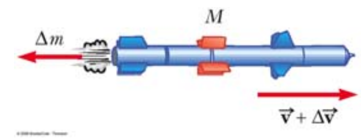


Rocket Propulsion, "initial" state



- The initial mass of the rocket is M + Δm
  - M is the mass of the rocket
  - m is the mass of the fuel
- The initial velocity of the rocket is $\vec{v}$
- The speed of the fuel is v_e relative to the rocket

Rocket Propulsion, "final" state



- The rocket's mass is M
- The mass of the fuel, Δm, has been ejected, with speed $v - v_e$ relative to the Earth
- The rocket's speed has increased to $\vec{v} + \Delta\vec{v}$

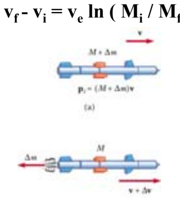
Rocket Propulsion, momentum conservation

$(M + \Delta m) v = M (v + \Delta v) + \Delta m (v - v_e)$

$M \Delta v = v_e \Delta m$

And $\Delta m = - \Delta M$, **then:** $M \Delta v = -v_e \Delta M$

$v_f - v_i = v_e \ln (M_i / M_f)$



Problem Solving for Collisions

- **Coordinates:** Set up coordinate axes and define your velocities with respect to these axes
  - It is convenient to choose the x- or y- axis to coincide with one of the initial velocities
- **Draw:** In your sketch, draw and label all the velocities and masses

Problem Solving for Collisions, 2

- **Conservation of Momentum:** Write expressions for the x and y components of the momentum of each object before and after the collision
- Write expressions for the total momentum before and after the collision in the x-direction and in the y-direction
- **Conservation of Energy:** If the collision is elastic, write an expression for the total energy before and after the collision

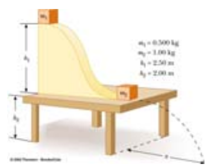
Problem Solving for Collisions, 3

- **Solve** for the unknown quantities
  - Solve the equations simultaneously
  - There will be two equations for inelastic collisions
  - There will be three equations for elastic collisions

Example Problems

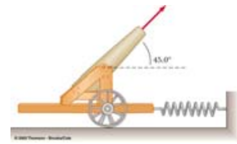
30. An 8.00-g bullet is fired into a 250-g block that is initially at rest at the edge of a table of height 1.00 m (Fig. P6.30). The bullet remains in the block, and after the impact the block lands 2.00 m from the bottom of the table. Determine the initial speed of the bullet.

57. A 0.500-kg block is released from rest at the top of a frictionless track 2.50 m above the top of a table. It then collides elastically with a 1.00-kg block that is initially at rest on the table, as shown in Figure P6.57. (a) Determine the velocities of the two blocks just after the collision. (b) How high up the track does the 0.500-kg block travel back after the collision? (c) How far away from the bottom of the table does the 1.00-kg block land, given that the table is 2.00 m high? (d) How far away from the bottom of the table does the 0.500-kg block eventually land?



More Problems

61. A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially unstretched and with force constant *k* = 2.00 x 10^4 N/m, as in Figure P6.61. The cannon fires a 200-kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal. (a) If the mass of the cannon and its carriage is 5 000 kg, find the recoil speed of the cannon. (b) Determine the maximum extension of the spring. (c) Find the maximum force the spring exerts on the carriage. (d) Consider the system consisting of the cannon, carriage, and shell. Is the momentum of this system conserved during the firing? Why or why not?



65. A block of mass *m* lying on a rough horizontal surface is given an initial velocity of v_0 . After traveling a distance *d*, it makes a head-on elastic collision with a block of mass 2*m*. How far does the second block move before coming to rest? (Assume the coefficient of friction μ_k is the same for both blocks.)

Summary of Chapter 6

Impulse is product of force and duration

Linear momentum is defined as product of mass and velocity

Impulse-momentum theorem relates the two.

Linear momentum for a closed system is conserved.

Elastic and inelastic collisions.

Rocket propulsion.