Chapter 13: Vibrations and Waves

Simple Harmonic Motion

When the restoring force has the mathematical form given by \( F = -kx \) (Hooke’s Law), the resultant periodic motion is referred to as “simple harmonic motion.”

Amplitude, \( A \), is the maximum position of the object relative to the equilibrium position.

The period, \( T \), is the time that it takes for the object to complete one complete cycle of motion, from \( x = A \) to \( x = -A \) and back to \( x = A \).

The frequency, \( f \), is the number of complete cycles or vibrations per unit time \( f = 1/T \).

\[
E_{\text{Total}} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2
\]

\[
v = \pm \sqrt{\frac{k}{m}} (A^2 - x^2)
\]

We already knew an example of SHM:

Uniform Circular Motion!

If we analyze the x-direction (or y-direction) motion of an object in uniform circular motion, we see that the centripetal force on the object is proportional to its displacement from the center of the circle in the x-direction. In other words, the projection of an object in uniform circular motion into one dimension creates a situation which is identical to an object bound to the end of a spring in a simple harmonic oscillation.
Kinematics of Simple Harmonic Motion

Since the x-component of the force on an object in uniform circular motion is the same as (the x-component of) the force on an object at the end of a spring. Both motions (in the x-direction) must be identical. To describe the motion of an object on a spring, we only need to analyze the 1-D projection of a object in uniform circular motion.

\[
x = A \cos \theta = A \cos(\omega t + \theta_0)
\]

\[
v_x = -A \omega \sin \theta = -A \omega \sin(\omega t + \theta_0)
\]

\[
a_x = -A \omega^2 \cos \theta = -A \omega^2 \cos(\omega t + \theta_0)
\]

Plugging these expressions into our original equation of \( F = -k x = m a \), we identify that

\[
\omega = \sqrt{\frac{k}{m}}
\]

\[
T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}
\]

\[
f = \frac{\omega}{2\pi}
\]

Note: Frequency of SHM is independent of amplitude.

Simple Harmonic Motion, Sinusoidal curves
Example Problem

A spring of negligible mass stretches 3.00 cm from its relaxed length when a force of 7.50 N is applied. A 0.500-kg particle rests on a frictionless horizontal surface and is attached to the free end of the spring. The particle is pulled horizontally so that it stretches the spring 5.00 cm and is then released from rest at $t = 0$. (a) What is the force constant of the spring? (b) What are the angular frequency $\omega$, the frequency, and the period of the motion? (c) What is the total energy of the system? (d) What is the amplitude of the motion? (e) What are the maximum velocity and the maximum acceleration of the particle? (f) Determine the displacement $x$ of the particle from the equilibrium position at $t = 0.500$ s.

The pendulum

The gravitational force acting on a mass $m$ on a string with length of $L$, when it is displaced a small distance $s$ from its equilibrium position, has an approximate tangential component of

$$F_t = -(mg/L) s,$$

acting in a direction to push the mass back toward the equilibrium position. So the force acting on the pendulum is the same as that from a spring with a spring constant of $k = (mg/L)$.

In general, the motion of a pendulum is not simple harmonic. However, for small angles (usually $< 15^\circ$), it becomes simple harmonic.
Motion of The Pendulum

Since the restoring force on the pendulum is proportional to the displacement of the pendulum, the resultant motion is obviously “simple harmonic”. The angular frequency of the pendulum motion is

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

T is independent of m!

More examples of simple harmonic motion

As long as you can establish that a restoring force acting on an object can be expressed in the form of $F = -kx$, the object is in a simple harmonic motion, with $\omega = (k/m)^{1/2}$ and $T=2\pi/\omega$. 
Waves: A wave is a traveling disturbance. It carries energy but involves no net transportation of material.

- Mechanical waves require
  - Some source of disturbance
  - A medium that can be disturbed
  - Some physical connection between or mechanism through which adjacent portions of the medium influence each other

Types Of Waves

- A transverse wave has disturbance perpendicular to the direction of the wave.
- A longitudinal wave has disturbance parallel to the direction of the wave.
### Description of a wave

**period** ($T$)  

**frequency** ($f = T^{-1}$)  

**amplitude** ($A$)  

**wavelength** ($\lambda$)  

The speed on a wave stretched under tension $F$ is  

$$v = \sqrt{\frac{F}{\mu}} \quad \text{where} \quad \mu = \frac{m}{l}$$  

$\mu$ is called the linear density  

$$\text{speed} \quad (v = \frac{\lambda}{T} = \lambda f)$$

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### Interference of Waves

**The Principle of Linear Superposition**

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.

\((a)\) At a particular time  

\((b)\) At a particular location  

\((c)\) At a particular time
Reflection of Waves

- When a traveling wave reaches a boundary, all or part of it is reflected.
- When reflected from a free end, the pulse is not inverted.
- When it is reflected from a fixed end, the wave is inverted.

Chapter 13 Summary

Simple harmonic motion occurs when the net force is proportional to the displacement and in the opposite direction.

Two examples: mass attached to spring and simple pendulum.
Transverse wave and longitudinal wave.
Amplitude, period, frequency, and wavelength of a periodic wave.
Speed of wave on a string.
Superposition principle of waves.
Reflection of waves.