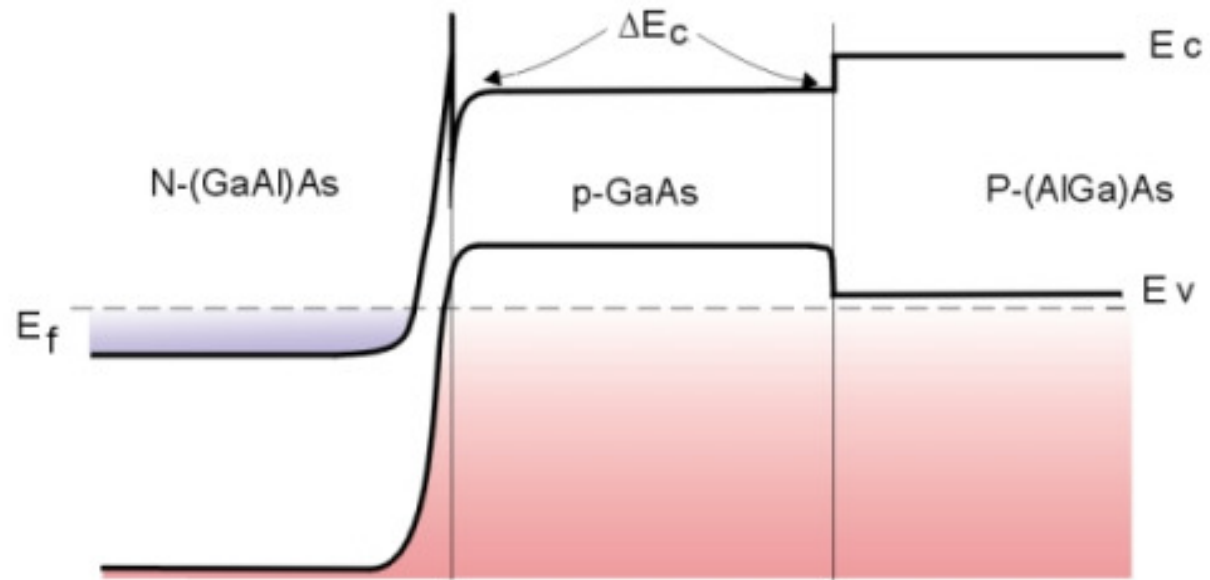


(a) Equilibrium bandstructure



(b) Forward biased

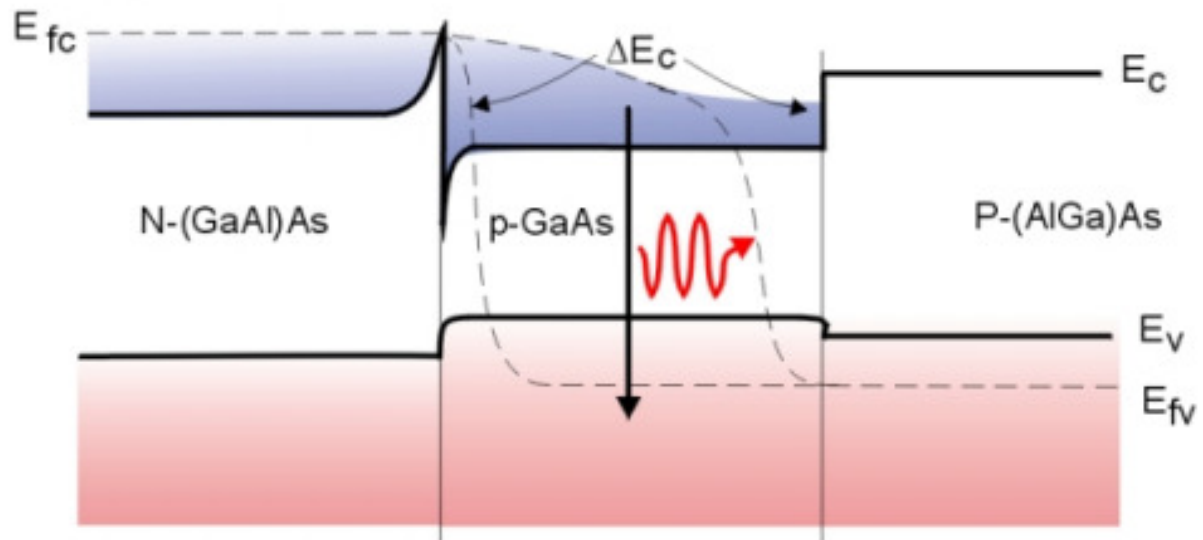
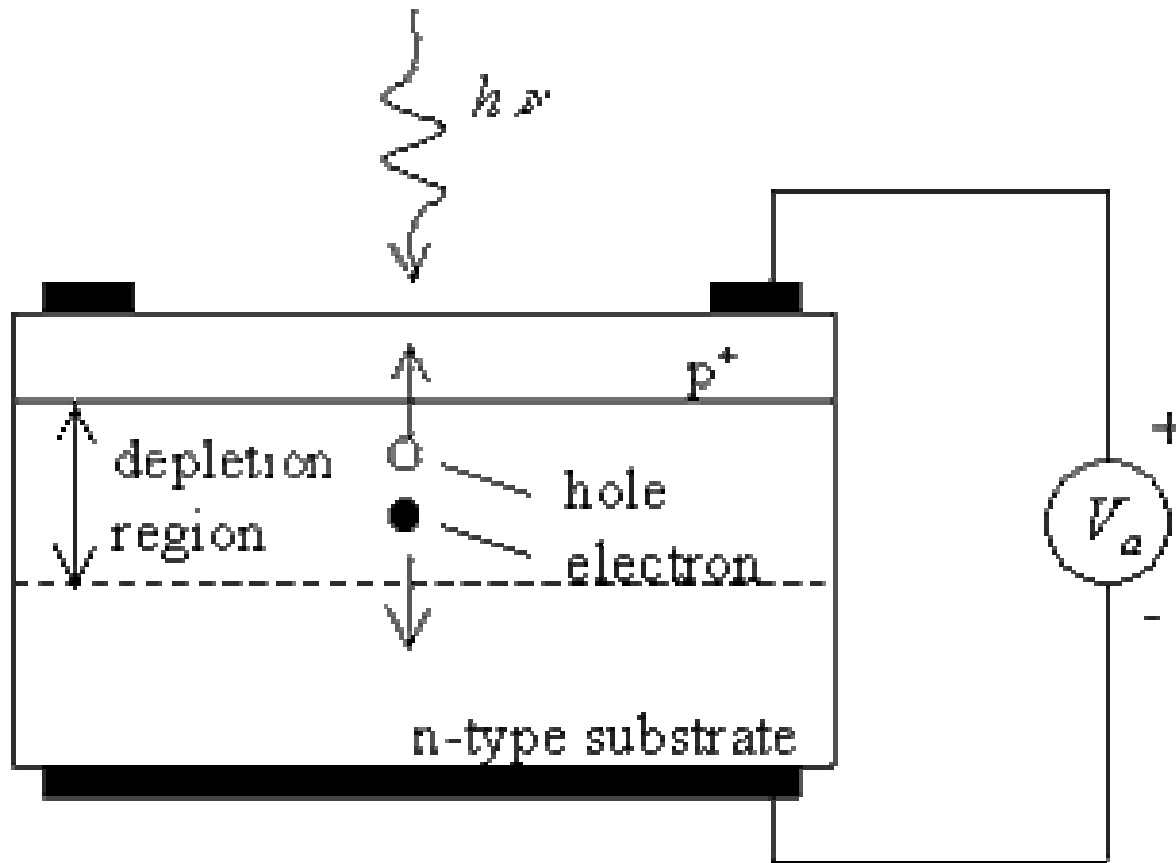


Photo Diode



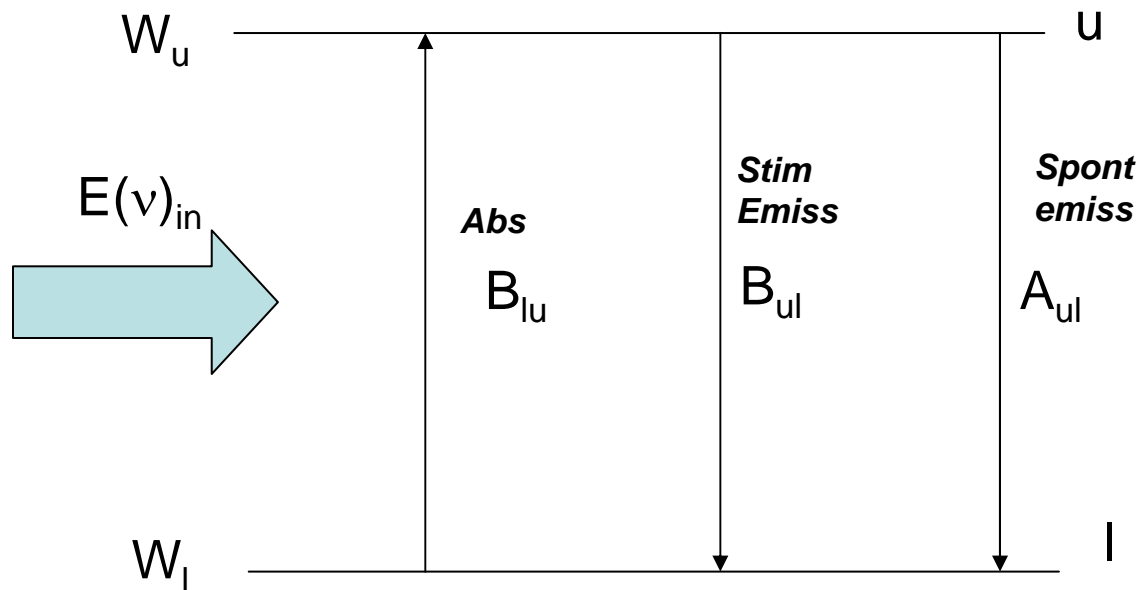
$$I = I_s (e^{V_a/V_T} - 1) - I_{ph}$$

$$I_{ph, \max} = \frac{q}{h\nu} P_{in}$$

$$I_{ph} = (1 - R)(1 - e^{-\alpha d}) \frac{q P_{in}}{h\nu}$$

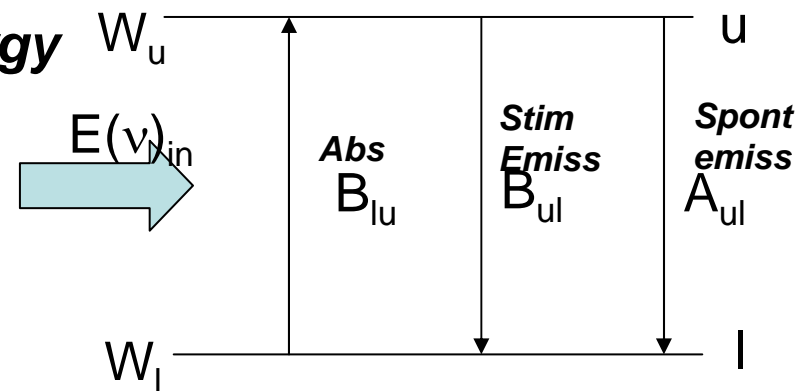
Interaction of Light & Atomic Systems

- **Assume**
 - **Only two possible states of energy: W_u and W_l**
 - **Energy levels are infinitesimally sharp**
 - **Optical transitions occur between u and l**
 - **Monochromatic radiation is emitted or absorbed at frequency ν_{ul}**



Interaction of Light & Atomic Systems - 2

- **The initial populations of the levels are N_u and N_l atoms/unit volume, respectively**
 - **The total population of the system**
 - $N_{tot} = N_l + N_u$
 - **Relative population given by Boltzmann**
 - $N_u/N_l = \exp[-h\nu_{ul}/k_B T]$
- **Spectral distribution of radiation within the enclosure characterised by atomic transition processes**
 - **absorption of energy (stimulated)**
 - **spontaneous emission of energy**
 - **stimulated emission of energy**



Interaction of Light & Atomic Systems

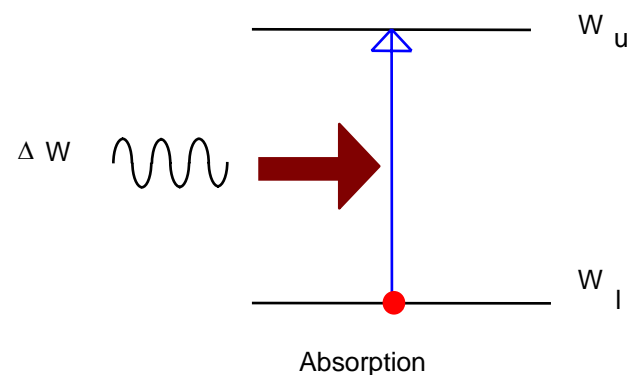
- ***At equilibrium the spectral radiation density within an isolated thermal cavity is constant and given by Planck's black-body relationship***

$$\rho(\nu) = \frac{(8\pi\nu^2 n^3 / c^3) h\nu}{[\exp(h\nu/k_B T) - 1]} \quad [\text{J Hz}^{-1} \text{ m}^{-3}]$$

- ***The number of oscillation modes per unit volume is $8\pi\nu^2 n^3 / c^3$***
- ***The photon energy is $h\nu$***
- ***The number of photons in a mode is $1/[\exp(h\nu/k_B T) - 1]$***
- **Energy Transition Processes**
 - **Above system is now irradiated by a monochromatic wave of radiation density $\rho(\nu)$ and frequency ν_{ul}**
 - **For each transition mechanism, simple rate equations describe atomic populations**

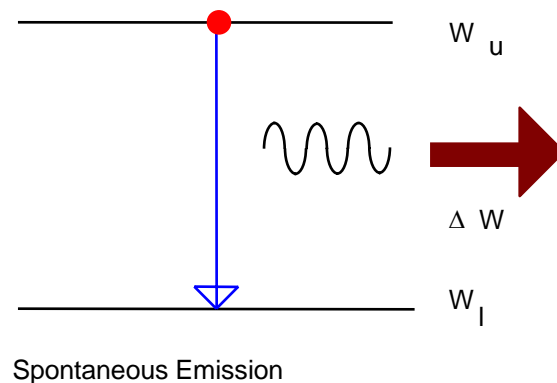
Stimulated Absorption

- Photons, of frequency ν_{ul} , from incident radiation are absorbed by the gas
 - Atoms are excited from level l to level u
- Rate at which lower level l is de-populated (loses atoms)
 - $(dN_l/dt)_{abs} = N_l \rho(\nu) B_{lu}$ $[s^{-1} m^{-3}]$
 - B_{lu} Einstein "B-coefficient" of absorption for the transition
 - proportional to the probability of a photon being absorbed between the levels
 - SI units of B_{lu} are $m^3 J^{-1} s^{-2}$



Spontaneous Emission

- ***Excited atoms can spontaneously de-excite from levels u to l***
- ***Photons are emitted by random process that occurs over a duration which is related to the natural lifetime of the energy transition***
 - ***phase and power of the emitted radiation is independent of any external radiation***

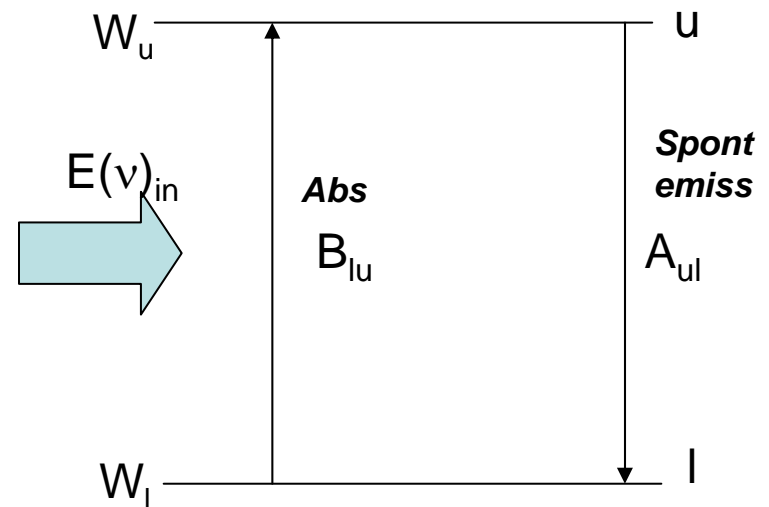


Spontaneous emission

- **Rate at which level u is depopulated**
 - $(dN_u/dt)_{\text{spont}} = A_{ul} N_u$ [s⁻¹ m⁻³]
 - A_{ul} Einstein "A-coefficient" of spontaneous emission
 - SI units of A_{ul} are s⁻¹
 - related to the probability of spontaneous emission between levels u and l
- **The spontaneous lifetime of the transition**
 - $\tau_{ul} = 1/A_{ul}$ [s]
 - Represents the average time an atom will reside in level u before de-excitation to level l
 - Typical values nanoseconds to seconds

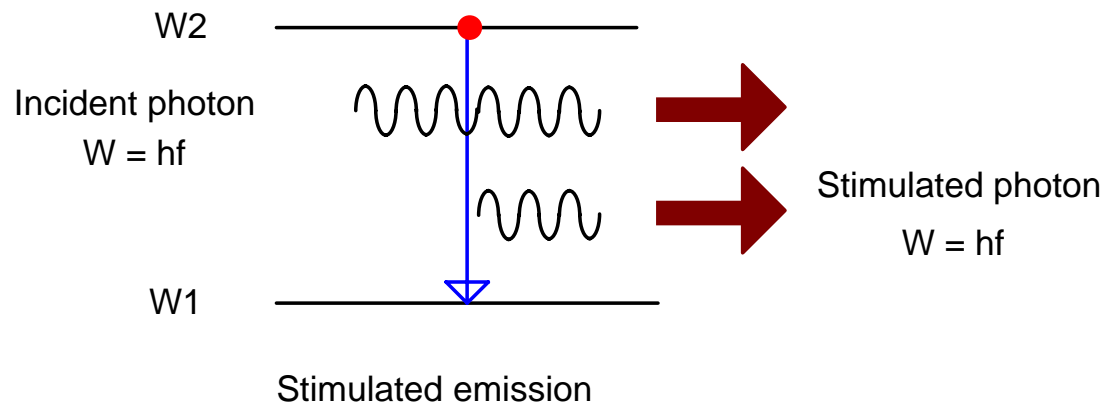
The Einstein Relationship – First Try

- System in thermal equilibrium with its surroundings
 - Relative population of levels at all times governed by the Boltzmann relationship
- For this to remain constant, the transition rate to the upper level must equal the transition rate to the lower level
 - $(dN_l/dt)_{abs} = (dN_u/dt)_{spont}$
- Thus
 - $[B_{lu} N_l \rho(\nu)] = [A_{ul} N_u]$
- Thus the radiation density is
 - $\rho(\nu) = (A_{ul}/B_{ul}) (N_l/N_u)$
 - $= (A_{ul}/B_{ul}) \exp[-h\nu_{ul}/k_B T]$
 - Doesn't Match Planck
 - $(8\pi\nu^2 n^3/c^3) h\nu$
 - $\rho(\nu) = \frac{\dots}{[\exp(h\nu/k_B T)-1]}$



Stimulated Emission

- Some excited atoms **induced** to de-excite from level u to l by **stimulated emission**
 - phase is identical to stimulating light
 - emitted power is proportional to incident power
- Hence the rate of de-population of level u is
 - $(dN_u/dt)_{stim} = B_{ul} N_u \rho(\nu)$ [$s^{-1} m^{-3}$]
 - B_{ul} is Einstein "B-coefficient" for stimulated emission
 - proportional to the probability of stimulated emission between u and l
 - SI units of B_{ul} are $m^3 J^{-1} s^{-2}$



The Einstein Relationship - 1

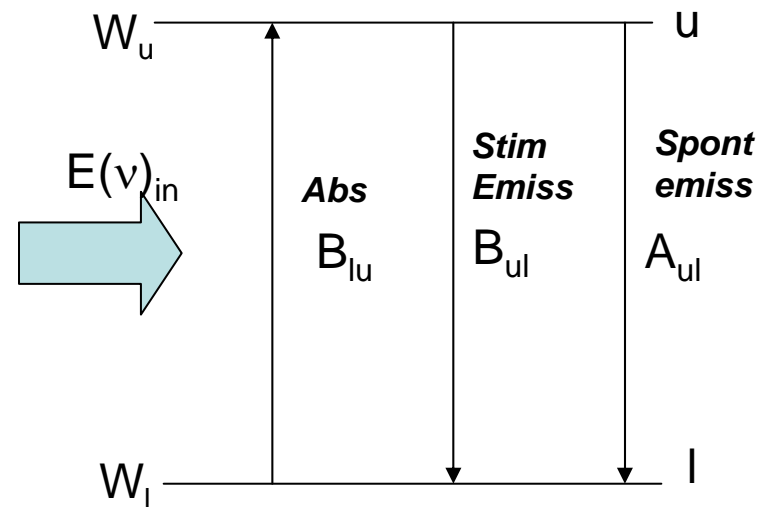
- $(dN_l/dt)_{\text{abs}} = (dN_u/dt)_{\text{spont}} + (dN_u/dt)_{\text{stim}}$
- Thus
 - $[B_{lu} N_l \rho(\nu)] = [A_{ul} N_u] + [B_{ul} N_u \rho(\nu)]$
- Thus the radiation density is

$$\rho(\nu) = \frac{(A_{ul}/B_{ul})}{(B_{lu}/B_{ul})(N_l/N_u)-1}$$

$$= \frac{(A_{ul}/B_{ul})}{(B_{lu}/B_{ul}) \exp[-h\nu_{ul}/k_B T]-1}$$

Matches Planck

$$\rho(\nu) = \frac{(8\pi\nu^2 n^3/c^3) h\nu}{[\exp(h\nu/k_B T)-1]}$$



The Einstein Relationship

- Also the ratio of spontaneous to stimulated emission coefficients is

$$\frac{A_{ul}}{B_{ul}} = \frac{8\pi h\nu^3 n^3}{c^3} \quad [\text{J Hz}^{-1} \text{ m}^{-3}]$$

- Thus we can rearrange for the stimulated emission coefficient,

$$B_{lu} = B_{ul} = B = \frac{c^3}{8\pi h\nu^3 n^3} A_{ul}$$

- We now express radiation density as

$$\rho(\nu) = \frac{A_{ul}}{B} \times \frac{1}{[\exp(h\nu/k_B T) - 1]} \quad \text{Einstein Relationship}$$

Significance of Stimulated Emission

- The ratio of stimulated to spontaneous output is

$$R = \frac{B_{ul}N_u\rho(\nu)}{A_{ul}N_u} = \frac{\rho(\nu)B_{ul}}{A_{ul}}$$

- On substituting for BB equn, $\rho(\nu)$, we get

$$R = 1/[\exp(h\nu/k_B T) - 1]$$

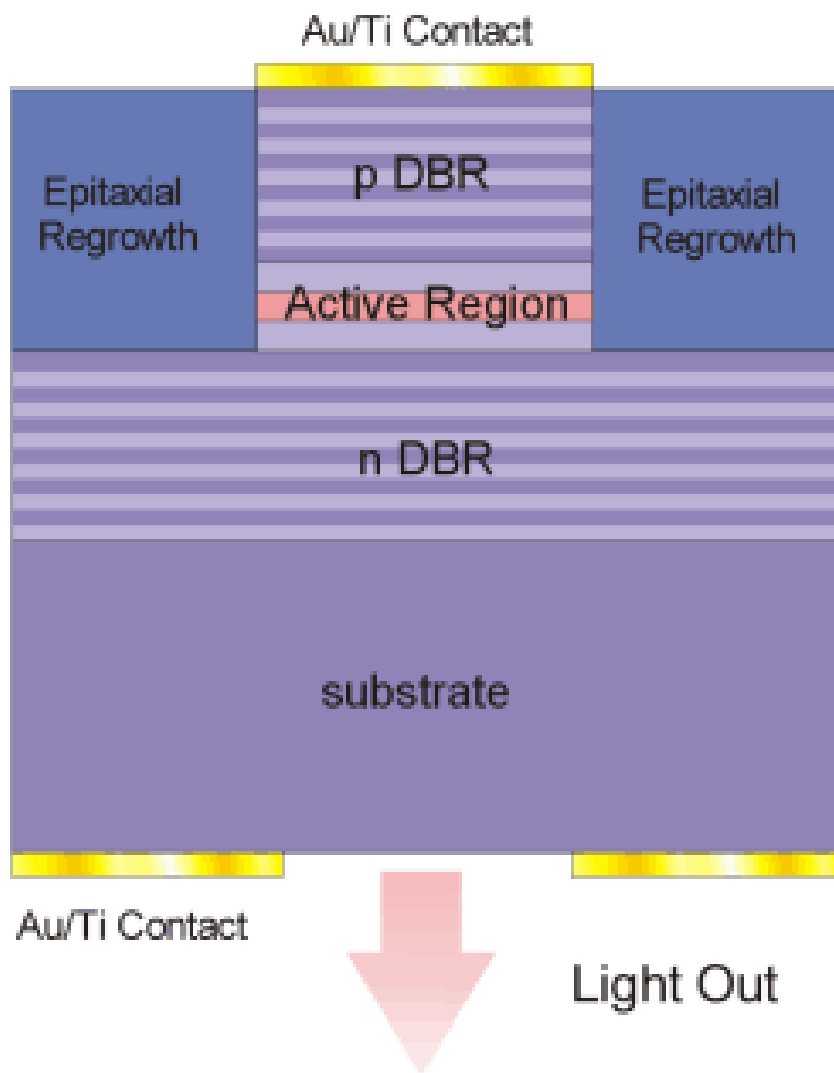
- For stimulated and spontaneous processes to be equally likely above equation should equal **unity**.
- Substituting some typical numbers will show that for this to occur naturally temperatures in excess of 30,000 K are needed!!

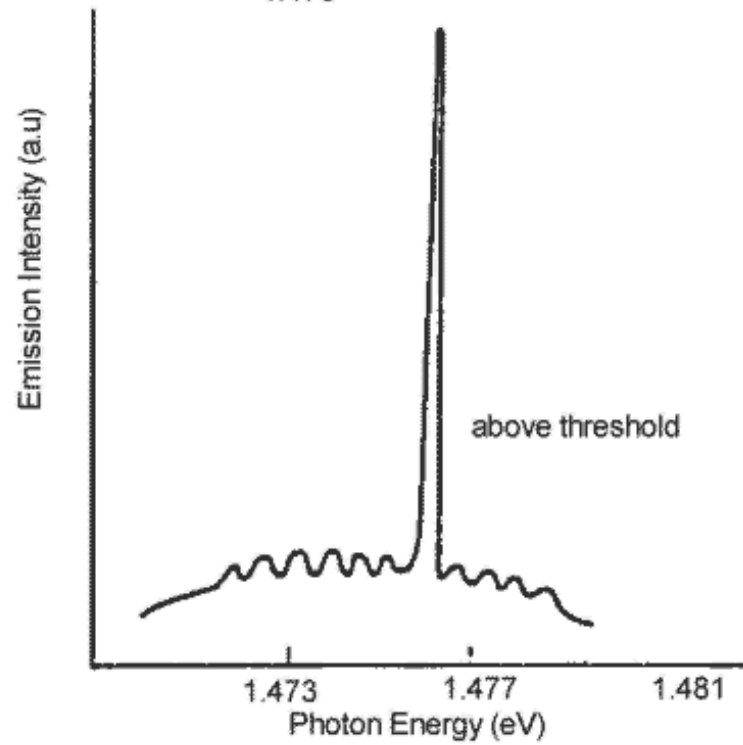
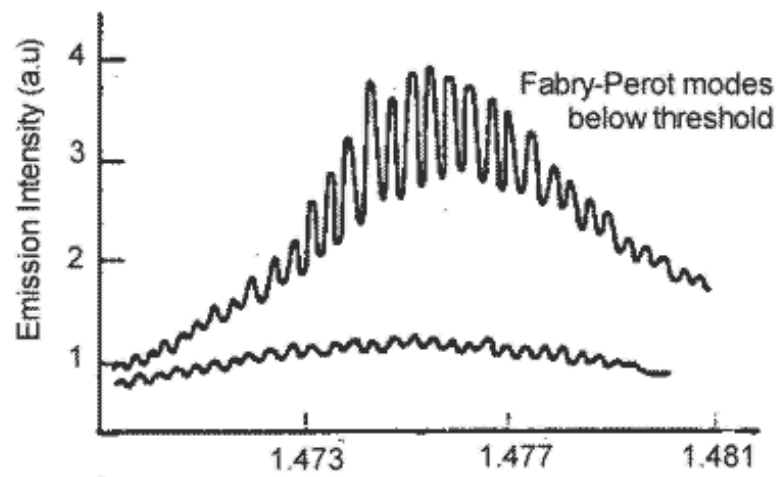
Significance of Stimulated Emission

- Have defined the no. of photons in a single mode earlier as

$$N_{ph}(m) = 1/[\exp(h\nu/k_B T) - 1]$$

- Then we see that *the number of photons in a given oscillation mode (cavity frequency) depends on the ratio of stimulated to spontaneous emission.*
- It is stimulated emission, which increases the number of photons in a given mode.
 - Spontaneous emission is a random process that bears no relationship to the incident beam in either phase or power.
 - Stimulated emission does bear a phase relationship to the incident beam and is proportional to the power of the incident beam.
- Hence if our aim is **to amplify light**
 - we must **promote stimulated emission** at the expense of spontaneous emission.
 - To increase the relative importance of stimulated emission, we need to **increase the radiation density within the cavity**.





Quantum Mechanics of Lasers

http://ocw.mit.edu/NR/rdonlyres/Chemistry/5-74Spring-2005/C4763150-57C5-469C-A8A2-9183E8A5FCAC/0/lec_9_10_abs_se.pdf

Prof. Andrei Tokmakoff, MIT Department of Chemistry

- **To verify the Einstein Relationship using quantum Mechanics, we must solve the Time dependent Schrödinger equation in the presence of a (perturbing) electric field (to represent the light).**

- **Wavefunction**

$$|\varphi\rangle = |\varphi_{EM}\rangle |\varphi_M\rangle$$

- **Fermi Golden Rule**

$$w_{k\ell} = \frac{2\pi}{\hbar} \delta(E_k - E_\ell - \hbar\omega) \left| \langle \varphi_F | V(t) | \varphi_I \rangle \right|^2$$

$$H = H_{EM} + H_M + V(t) = H_0 + V(t)$$

$$H_{EM} = \sum_{\vec{k}, j} \hbar\omega_{\vec{k}} \left(a_{\vec{k}j}^\dagger a_{\vec{k}j} + \frac{1}{2} \right)$$

$$H_M = \sum_i \frac{p_i^2}{2m_i} + V_i(\vec{r}, t)$$

$$V(t) = \frac{-q}{m} \vec{A} \cdot \vec{p}$$

$$= \sum_{\vec{k}, j} \frac{q}{m} \sqrt{\frac{2\pi\hbar}{v \omega_k}} (\hat{\epsilon}_j \cdot \vec{p}) \left[a_{\vec{k}, j} e^{i(\vec{k} \cdot \vec{r} - \omega t)} + a_{\vec{k}, j}^\dagger e^{-i(\vec{k} \cdot \vec{r} - \omega t)} \right]$$

$$= V^{(-)} + V^{(+)}$$

Quantum Mechanics of Lasers

- **Absorption Rate and Stimulated Emission Rate**

$$w_{k\ell}(\omega) = \frac{\pi}{2\hbar^2} |E_0(\omega)|^2 |\langle k | \hat{\epsilon} \cdot \bar{\mu} | \ell \rangle|^2 \delta(\omega_{k\ell} - \omega) = \frac{|E_0(\omega_{k\ell})|^2 \omega^2}{6\pi \hbar^2 c^3} |\bar{\mu}_{k\ell}|^2$$

$$B_{k\ell} = \frac{4\pi^2}{3\hbar^2} |\mu_{k\ell}|^2$$

- **Spontaneous Emission Rate**

$$W_{nm} = \frac{\hbar \omega^3}{\pi^2 c^3} B_{nm} (N + 1)$$

$$A_{nm} = \frac{\hbar \omega^3}{\pi^2 c^3} B_{nm}$$

<http://ece-www.colorado.edu/~bart/book/movie/movie5.htm>

Semiconductor Laser

Photo Diode

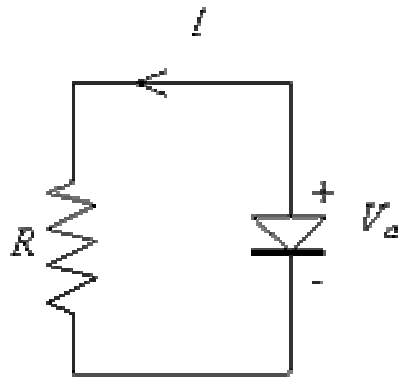
Optoelectronic Transmitter and Receiver

Silicon MOSFET

Digital Light Projector

Cell Phone

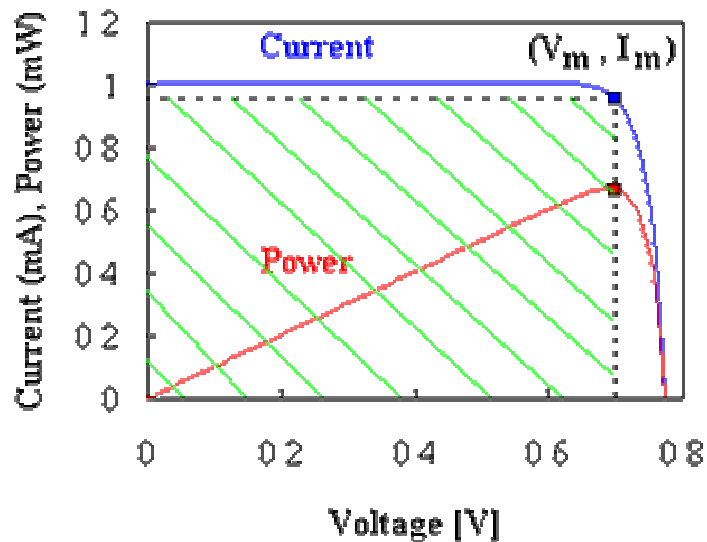
Solar Cell



A 1 cm² silicon solar cell has a saturation current of 10-12 A and is illuminated with sunlight yielding a short-circuit photocurrent of 25 mA. Calculate the solar cell efficiency.

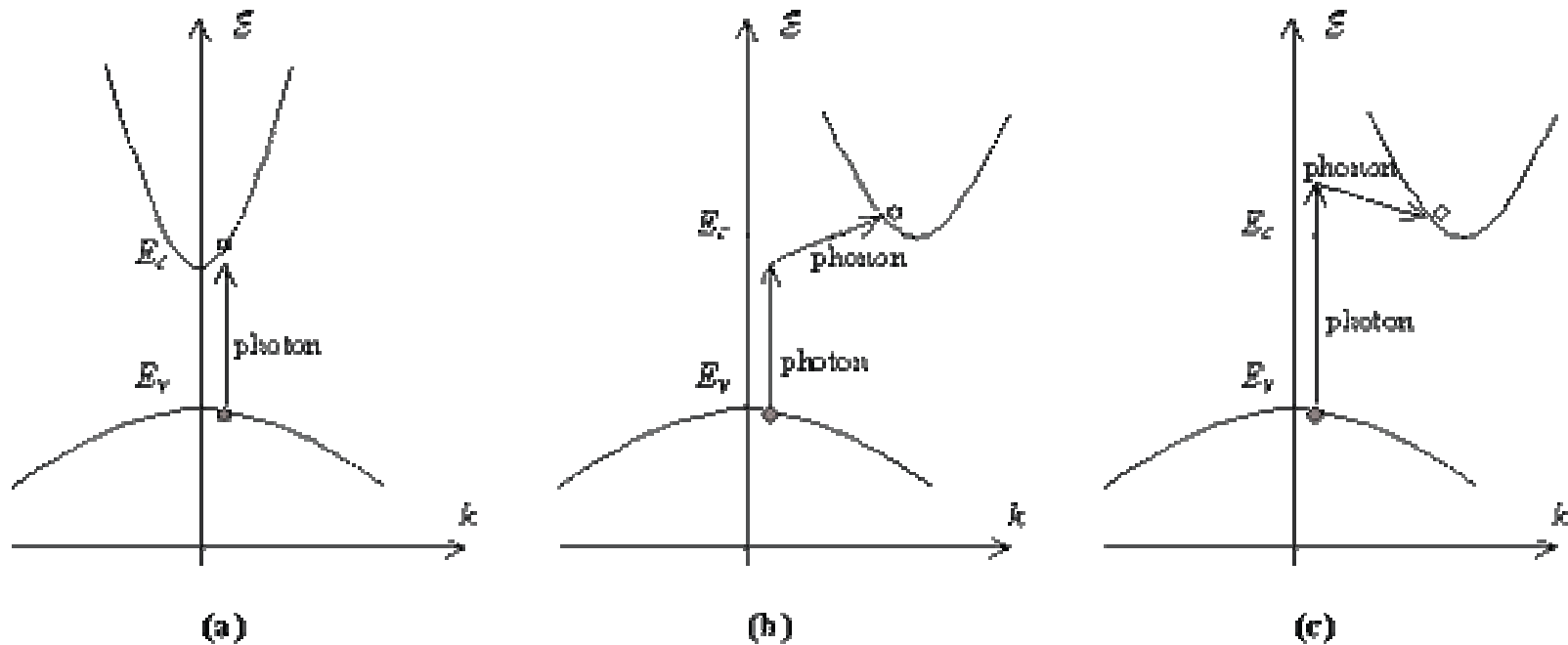
The maximum power is generated for:

$$\frac{dP}{dV_a} = 0 = I_s (e^{V_m/V_t} - 1) - I_{ph} + \frac{V_m}{V_t} I_s e^{V_m/V_t}$$



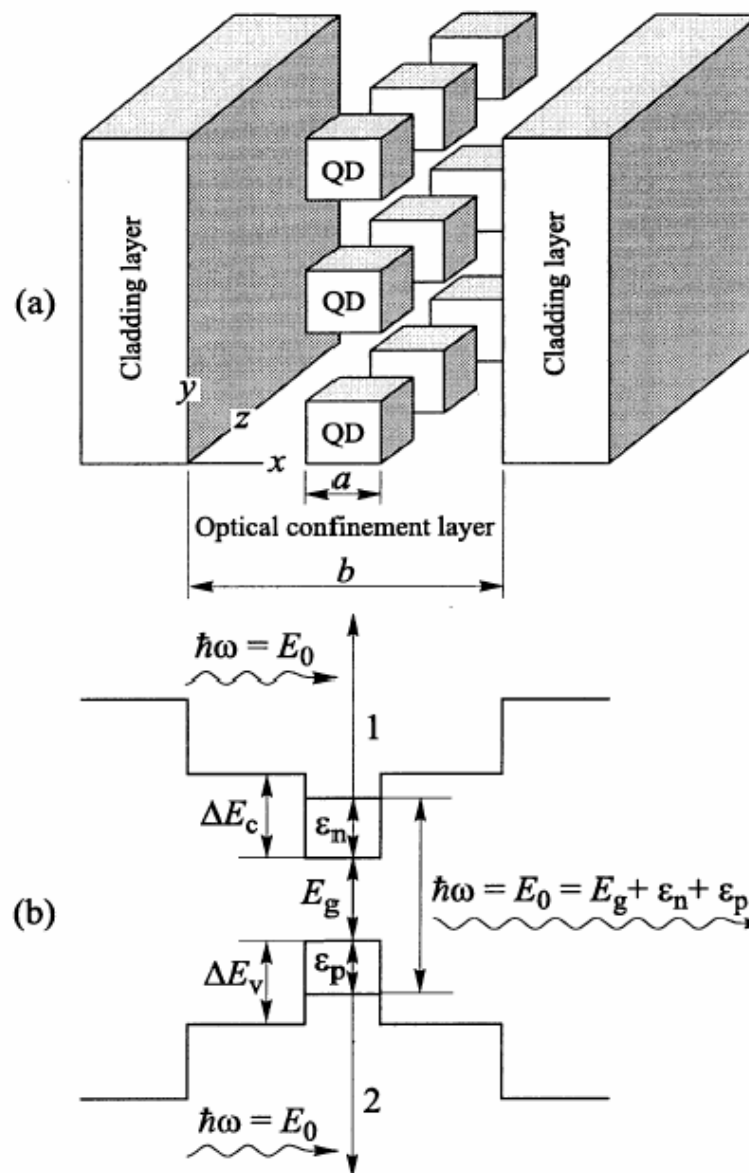
Using iteration $V_m = 0.540$ V and the efficiency equals:

$$\eta = \left| \frac{V_m I_m}{P_{in}} \right| = \frac{0.54 \times 0.024}{0.1} = 13\%$$



E - k diagram illustrating a) Photon absorption in a direct bandgap semiconductor b) Photon absorption in an indirect bandgap semiconductor assisted by phonon absorption and c) Photon absorption in an indirect bandgap semiconductor assisted by phonon emission.

Asryan and Suris, Int. J. High Speed Elec. and Sys. **12**, 111 (2002)



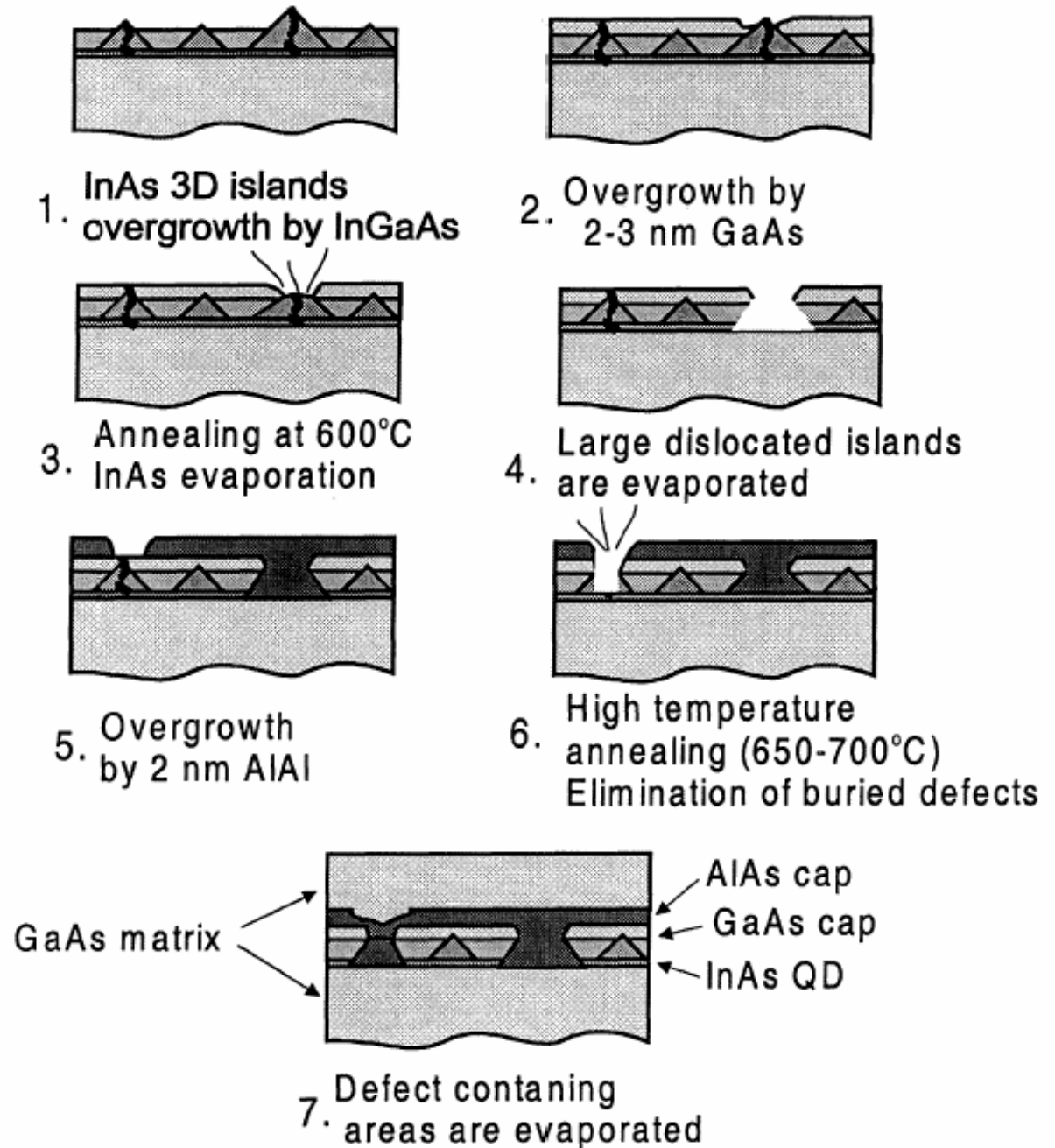
Issues:

Size Uniformity (etc.)
 Recombination
 Dot Density

Fig. 1. Schematic (a) and energy band diagram (b) of a QD laser structure. The QDs are not drawn to scale. Arrows 1 and 2 show the transitions of carriers from the quantized energy levels to the continuous-spectrum states in the process of light absorption.

Defect Reduction:

N.N. Ledentsov et al., Int. J. High Speed Elec. and Sys. **12**, 177 (2002)



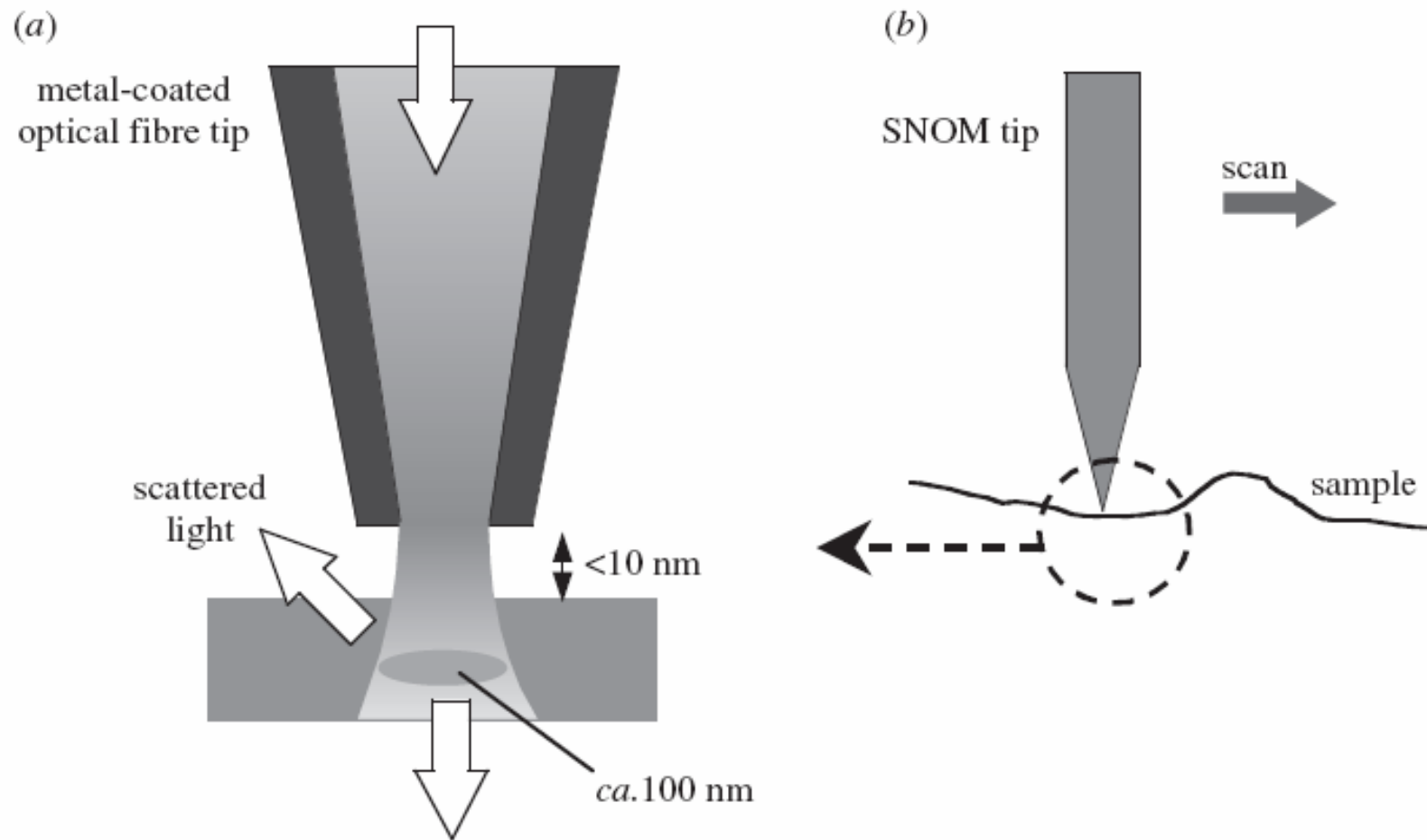
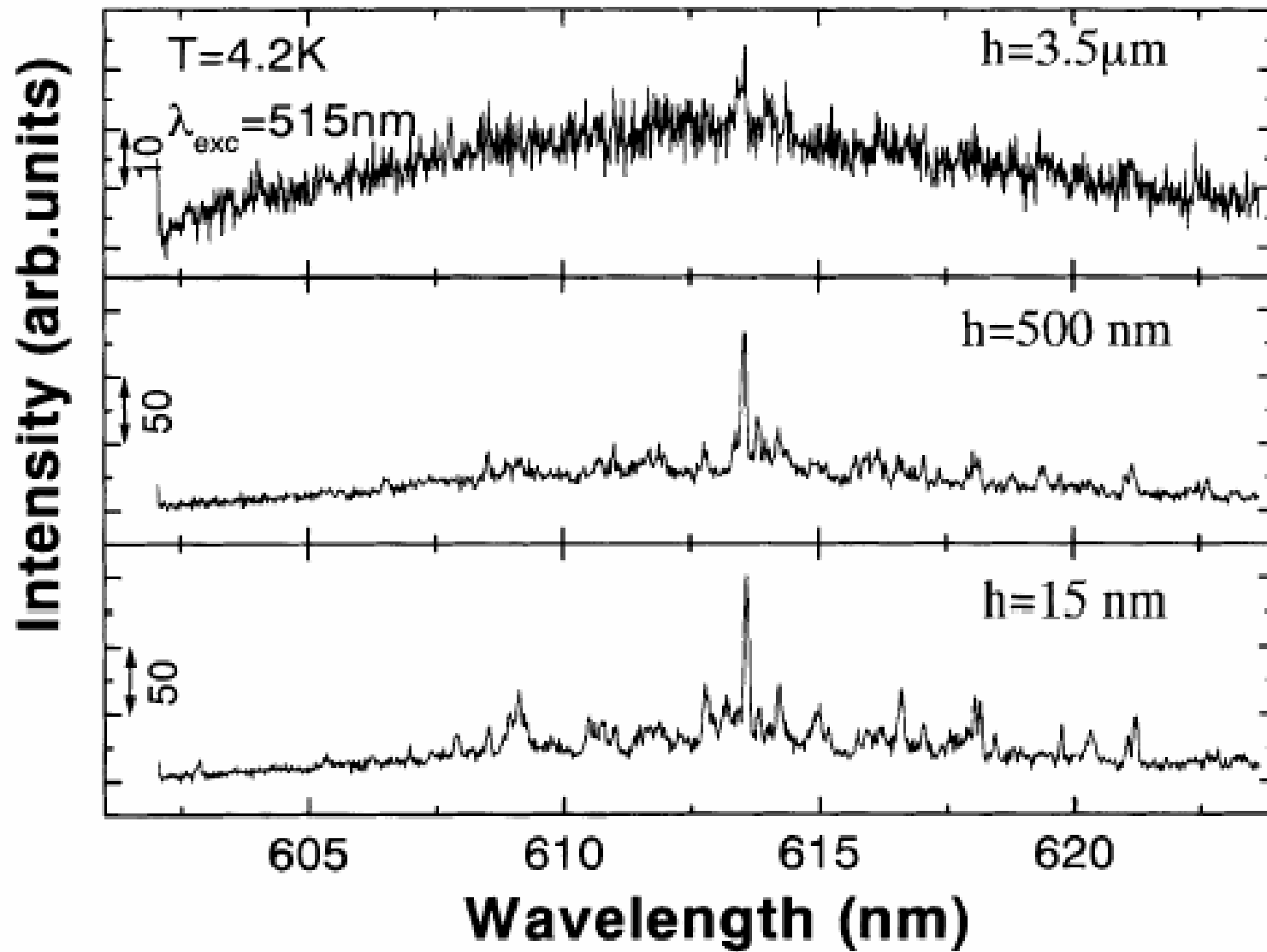


Figure 1. A schematic of aperture-SNOM. (b) An optical-fibre tip is scanned across a sample surface to form an image. The tip is coated with metal everywhere except at the apex, where a small aperture has been created. (a) It is held within a few nanometres of the sample surface so that a region less than 100 nm across is illuminated.

For general SNOM, see

D. Richards, *Phil. Trans. R. Soc. Lond. A* (2003) **361**, 2843–2857



M. Brun et. al, J. Microscopy **202**, 202 (2000)

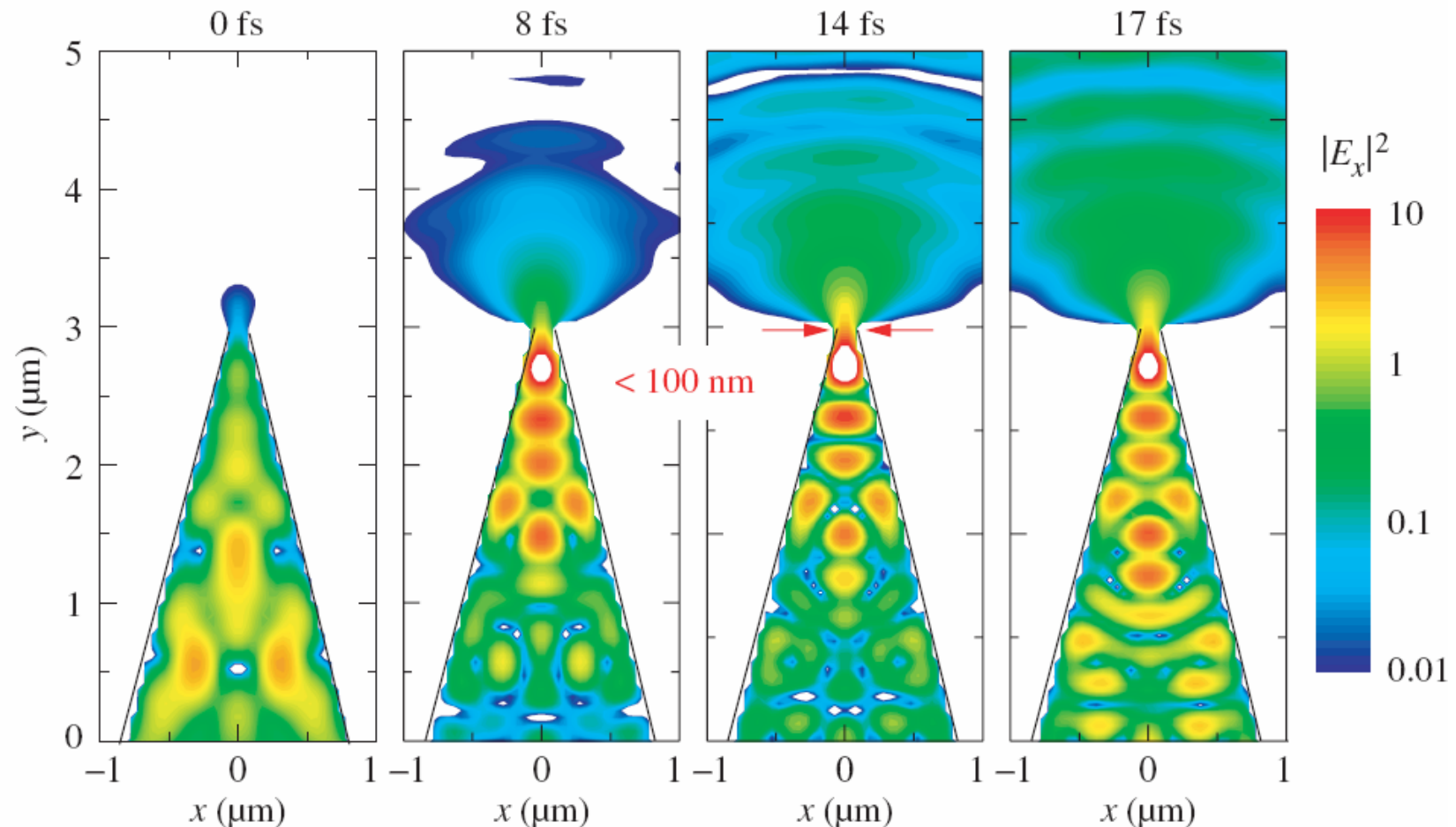


Figure 1. 2D FDTD simulation of the spatiotemporal evolution of a 10 fs light pulse at a wavelength of 810 nm propagating through a tapered, metalized fibre probe of 100 nm aperture diameter. The field intensity $|E_x(x, y, t)|^2$ is displayed on a logarithmic intensity scale at four different instants in time. Around $t = 14$ fs the pulse centre reaches the aperture, generating ultrashort near-field light spot directly below the aperture. Note the strong back reflection of the pulse inside the fibre. The metal coating is assumed to be a perfect conductor.

Christoph Lienau, *Phil. Trans. R. Soc. Lond. A* **362**, 861 (2004)

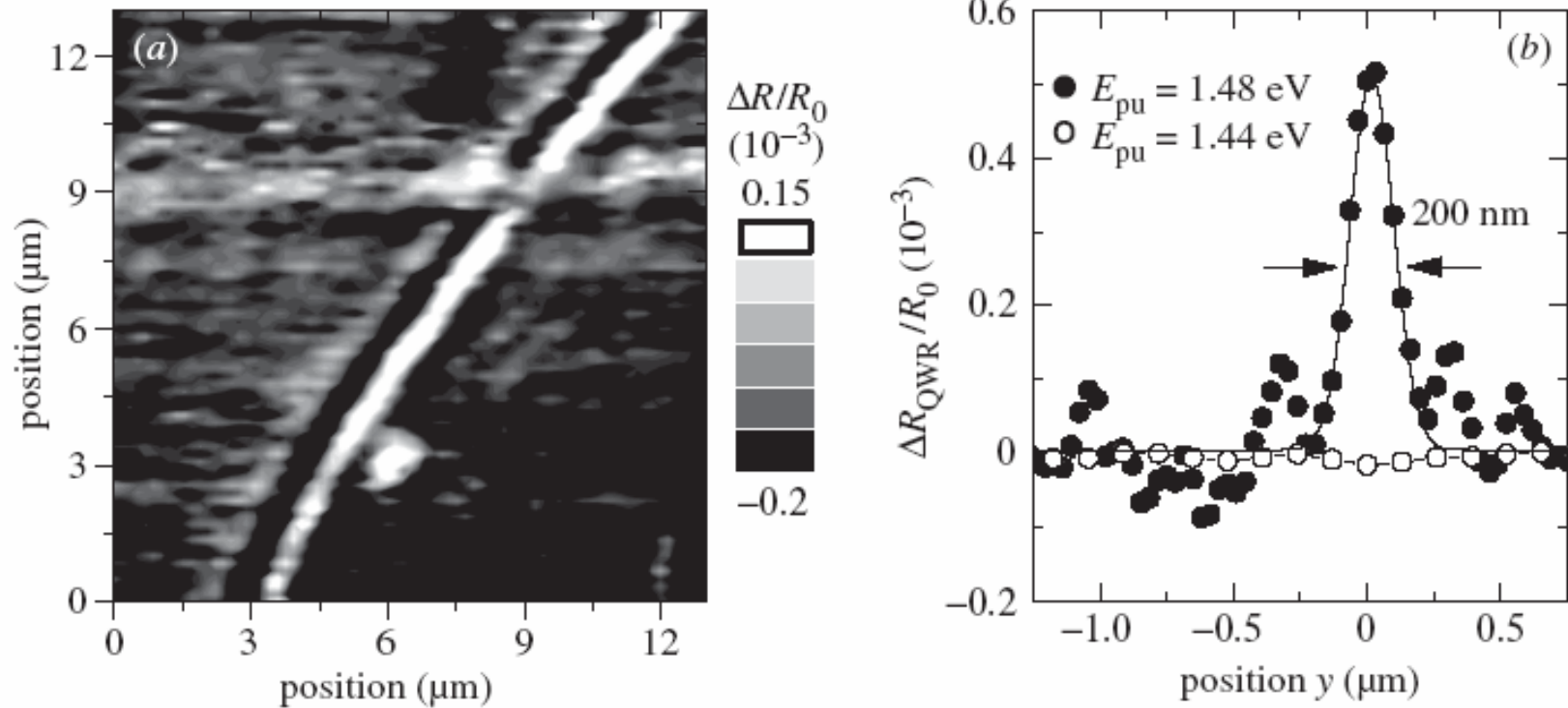


Figure 4. (a) Spatial map of the nonlinear pump-induced change in reflectivity of a single GaAs QWR. The map is recorded at room temperature with a probe laser set to 1.45 eV at the centre of the QWR absorption resonance. A pump laser at 1.52 eV generates electron-hole pairs in the QW surrounding the QWR. Trapping of carriers into the QWR bleaches the QWR absorption and decreases the reflectivity. (b) Spatial variation of the change in reflectivity along a line perpendicular to the QWR axis at a probe energy of 1.46 eV. The closed circles show data for resonant QWR excitation with a pump laser at $E_{\text{pu}} = 1.48$ eV, demonstrating 200 nm spatial resolution. For off-resonant excitation at $E_{\text{pu}} = 1.44$ eV the nonlinear signal vanishes (open circles).

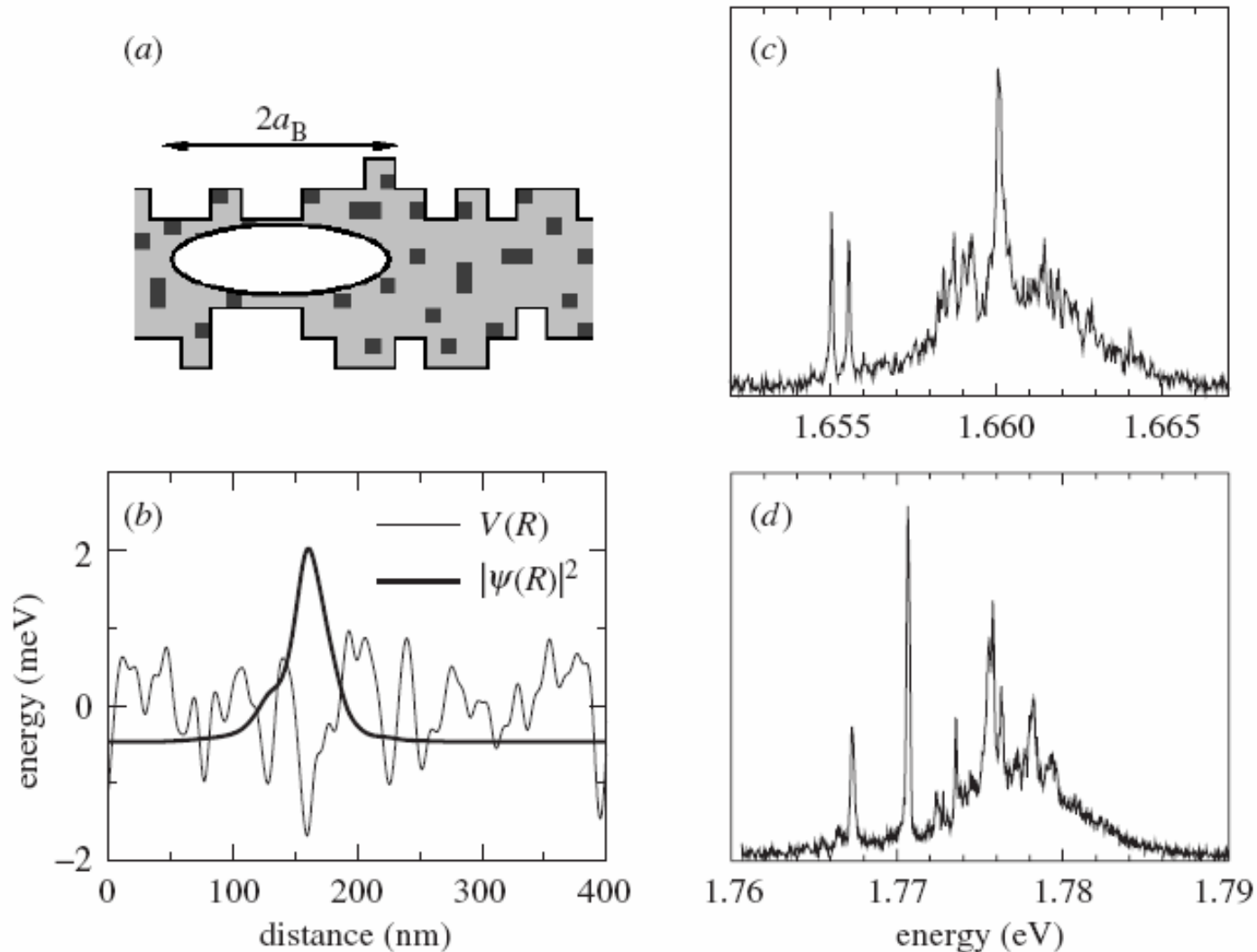


Figure 5. (a) Disorder in QWs arises from spatial fluctuations of the local QW thickness (interface roughness) and of the QW composition (alloy disorder). (b) Schematic of the effective disorder potential $V(\mathbf{R})$ and of a localized excitonic centre-of-mass wave function $|\psi(\mathbf{R})|^2$. Representative near-field PL spectra ($T = 12$ K) of (c) a 5.1 nm thick and (d) a 3.3 nm thick (100) GaAs QW.

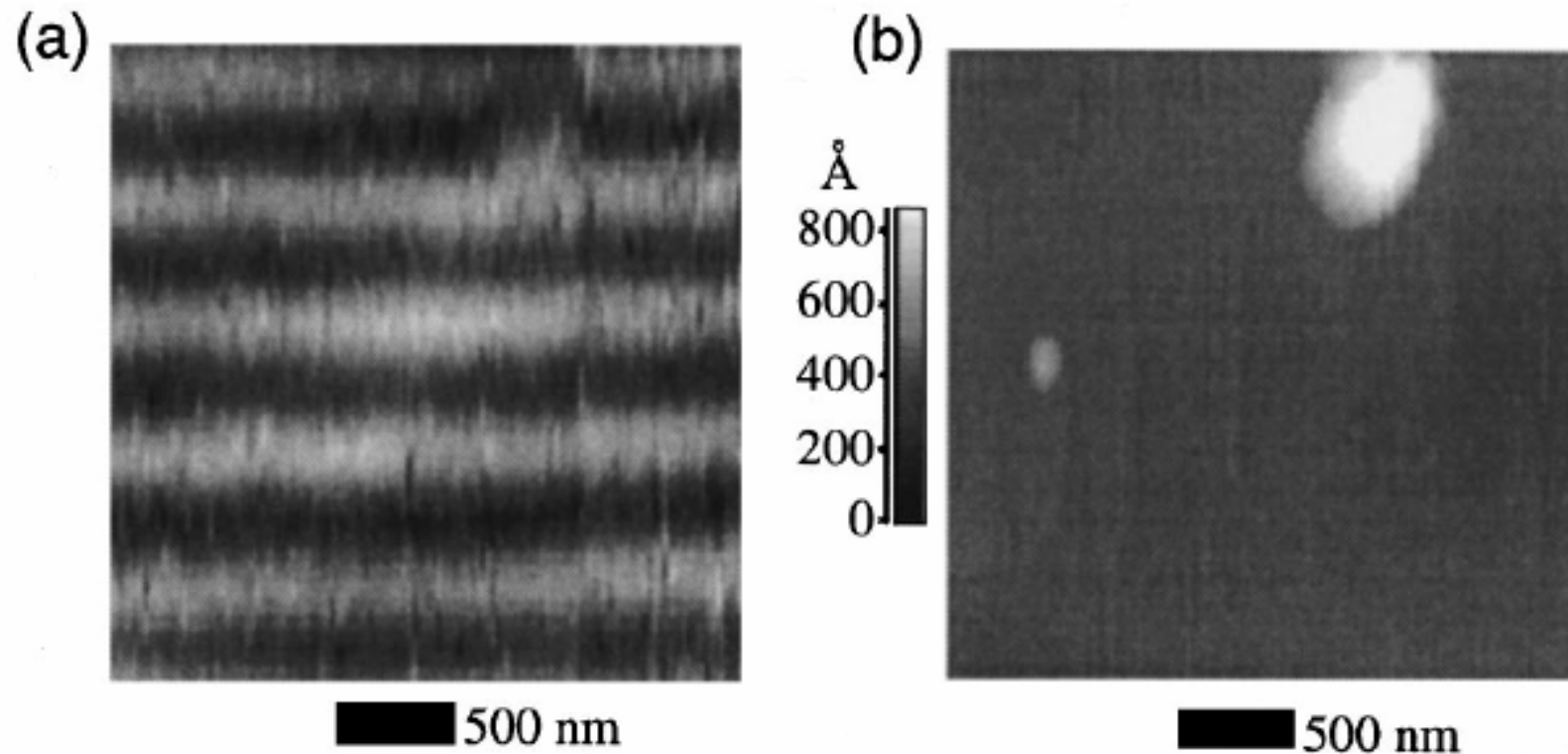
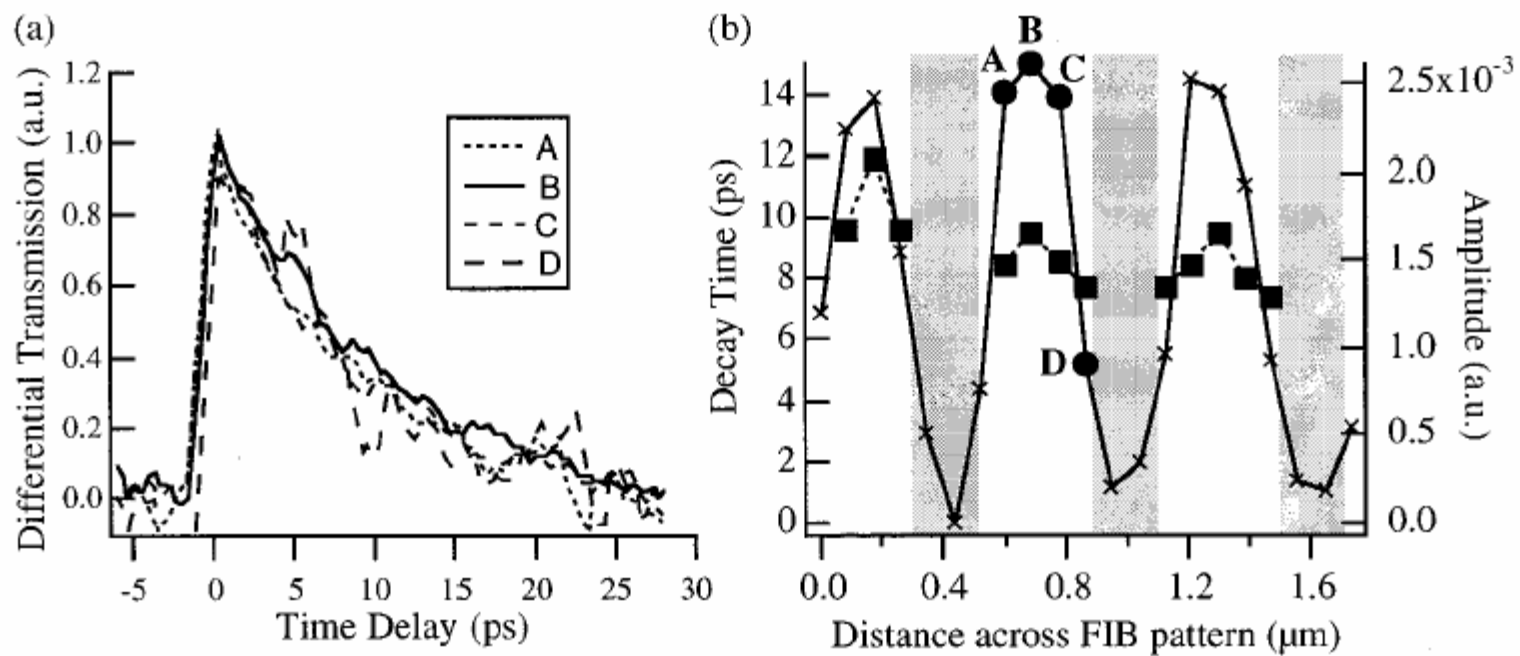
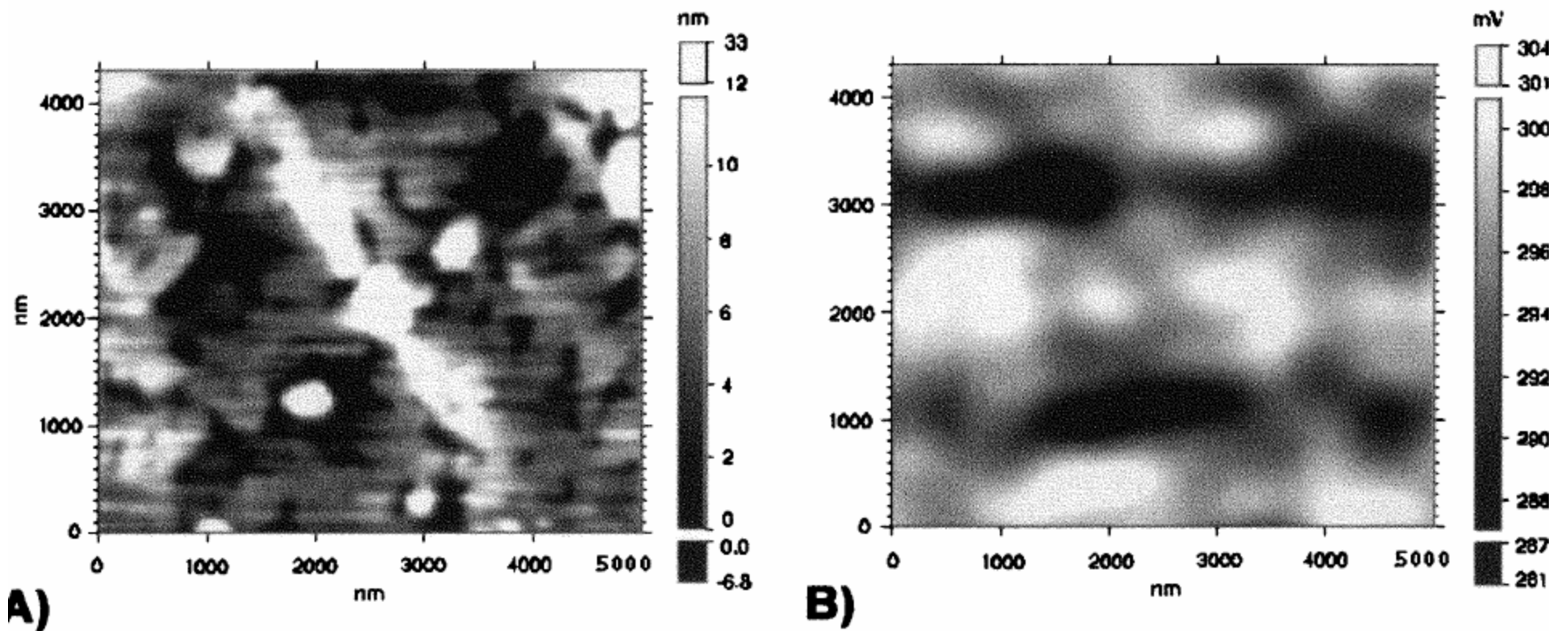


FIG. 5. Two-dimensional scans of pump-probe amplitude at zero time delay (a) and simultaneously measured topographical (b) for sample 1: GaAs/AlGaAs quantum well with 200 nm FIB implanted stripes and 400 nm spaces.

B. A. Nechay et. al, Rev. Sci. Instrum. **70**, 2758 (1999)



7. (a) Normalized pump-probe scans and (b) pump-probe amplitude at zero time delay (\times) and decay time from a single-exponential fit (\blacksquare) vs distance across the sample, for sample 1. The gray region corresponds to the implanted stripe.



4. Topography (A) and SNOM photoluminescence map at 2.25 eV (B) of the AlN-capped quantum dots sample. The cap layer shows the island-like features reported in Fig. 2. In the PL map a background signal arising from deeper QD layers is superimposed on the modulation related to the morphology of the last QD plane.

A. Zrenner, J. Chem. Phys.,
Vol. 112, No. 18, 8 May 2000

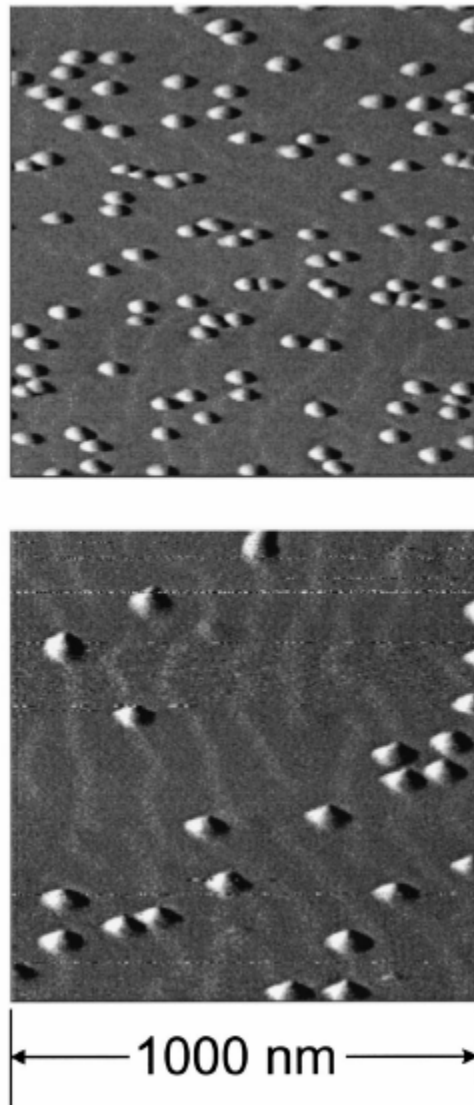


FIG. 1. AFM images of self assembled $\text{In}_{0.4}\text{Ga}_{0.6}\text{As}$ QDs grown by MBE on a GaAs substrate at a temperature of 530°C . Nominally 7.5 monolayers of

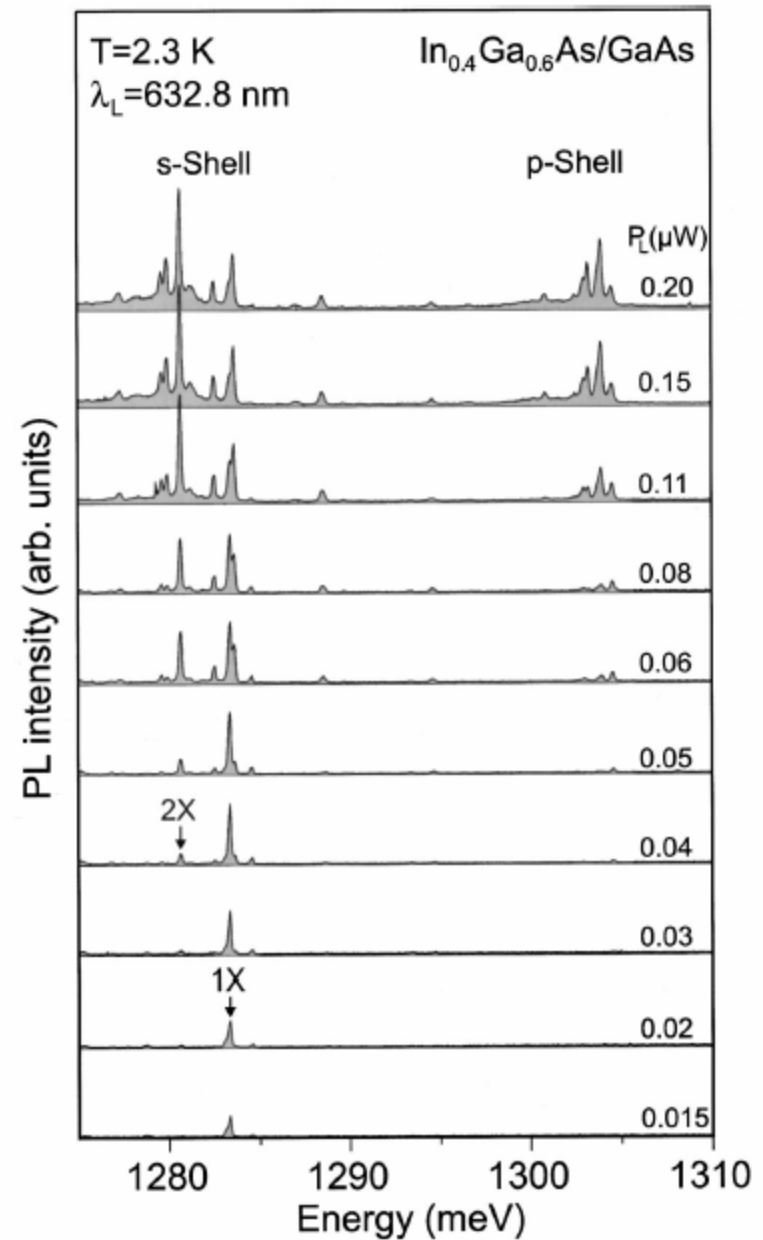


FIG. 2. Power dependent PL spectra from a single QD isolated by near field spectroscopy through a nano-aperture. At low excitation power P_L only the single exciton decay from the s -shell is observed (1X). At elevated P_L occupancies with two and more excitons are realized. The sequential biexciton decay leads to the appearance of the biexciton line (2 \rightarrow 1) in the