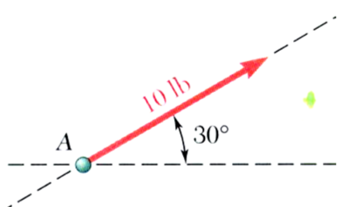
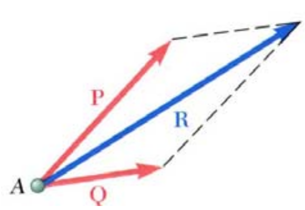
Chapter 2 Statics of Particles

- The effects of forces on particles:
  - replacing multiple forces acting on a particle with a single equivalent or *resultant* force,
  - relations between forces acting on a particle that is in a state of *equilibrium*.
- NOTE: The focus on “particles” does not imply a restriction to miniscule bodies. Rather, the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point. And, more importantly, we do not need to worry about rotation or torques (moments) of the system.

Resultant of Two Forces

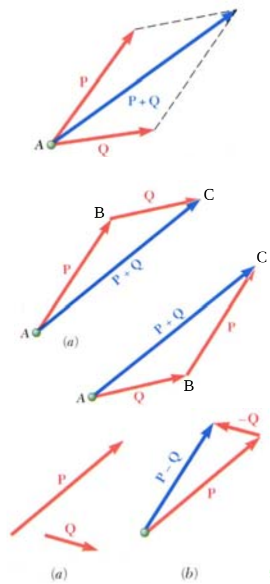


- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.



- The combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

Addition of Vectors



Vectors and scalars

- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

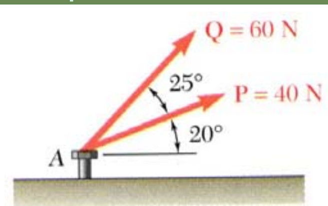
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

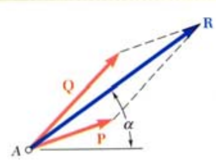
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Sample Problem 2.1

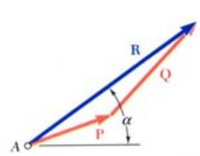


The two forces act on a bolt at *A*. Determine their resultant.



- Graphical solution - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

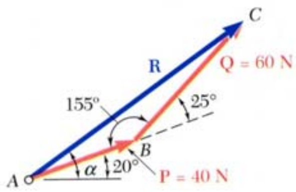
$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35°}$$



- Graphical solution - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\boxed{\mathbf{R} = 98\ \text{N} \quad \alpha = 35°}$$

Sample Problem 2.1



- Trigonometric solution - Apply the triangle rule.

  From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

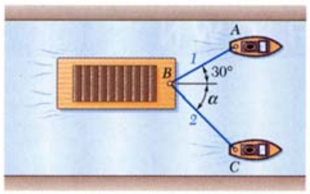
$$\boxed{R = 97.73\text{N}}$$

From the Law of Sines,

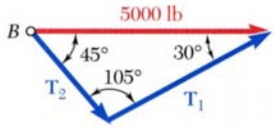
$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Sample Problem 2.2



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine the tension in each of the ropes for α = 45º.

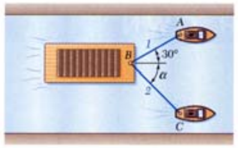


- Trigonometric solution - Triangle Rule with Law of Sines

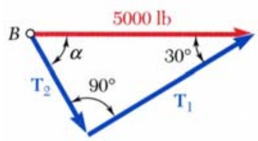
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

$$\boxed{T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}}$$

What if…?



At what value of α would the tension in rope 2 be a minimum?



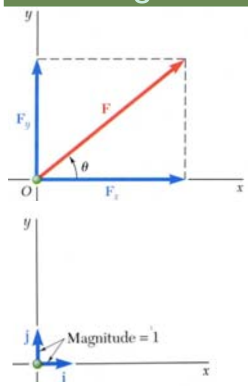
- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

$T_2 = (5000\,\text{lbf})\sin 30°$ $\qquad \boxed{T_2 = 2500\,\text{lbf}}$

$T_1 = (5000\,\text{lbf})\cos 30°$ $\qquad \boxed{T_1 = 4330\,\text{lbf}}$

$\alpha = 90° - 30°$ $\qquad \boxed{\alpha = 60°}$

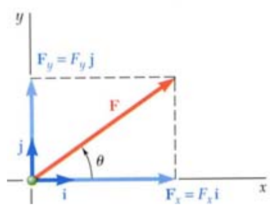
Rectangular Components of a Force: Unit Vectors



- It's possible to resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

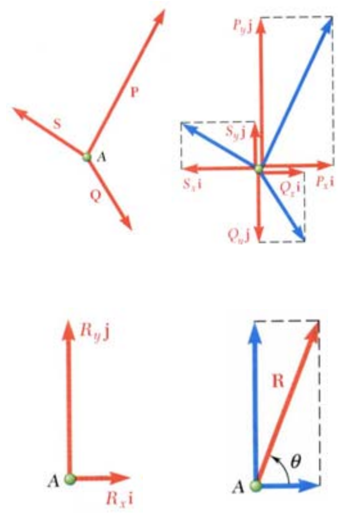


- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of Forces by Summing Components



- To find the resultant of 3 (or more) concurrent forces,

  $$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components, then add the components in each direction:

  $$R_x\vec{i} + R_y\vec{j} = P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j}$$
  $$= (P_x + Q_x + S_x)\vec{i} + (P_y + Q_y + S_y)\vec{j}$$

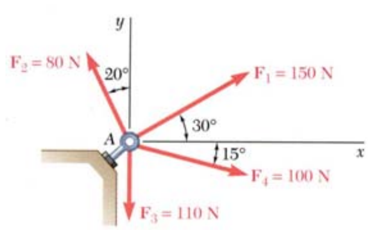
- The scalar components of the resultant vector are equal to the sum of the corresponding scalar components of the given forces.

  $$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

- To find the resultant magnitude and direction,

  $$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem 2.3



Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

- Resolve each force into rectangular components.

force	*mag*	*x – comp*	*y – comp*
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	–27.4	+75.2
$\vec{F}_3$	110	0	–110.0
$\vec{F}_4$	100	+96.6	–25.9
		$R_x = +199.1$	$R_y = +14.3$

- Determine the components of the resultant by adding the corresponding force components.

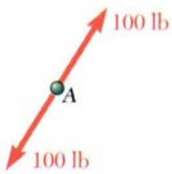
- Calculate the magnitude and direction.

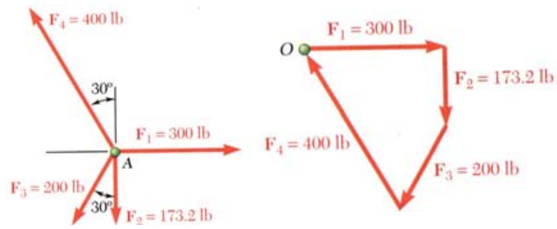
  $$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\,\text{N}}$$

  $$\tan\alpha = \frac{14.3\,\text{N}}{199.1\,\text{N}} \qquad \boxed{\alpha = 4.1°}$$

Equilibrium of a Particle

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.





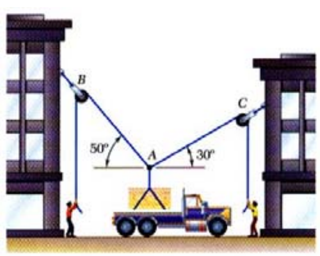
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense

- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

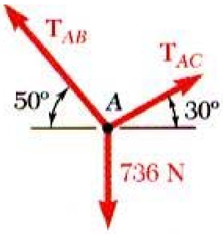
$$\boxed{\sum F_x = 0 \qquad \sum F_y = 0}$$

Free-Body Diagrams

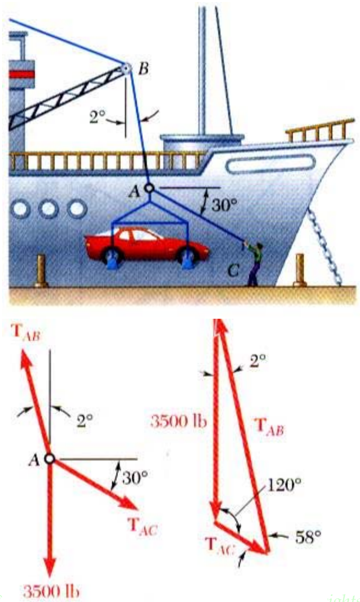


Free Body Diagram: A sketch showing only the forces on the selected particle. <u>This must be created by you</u>.

Space Diagram: A sketch showing the physical conditions of the problem, usually provided with the problem statement, or represented by the actual physical situation.



Sample Problem 2.4



In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

- Construct a free body diagram for the particle at *A*, and the associated polygon.
- Apply the conditions for equilibrium and solve for the unknown force magnitudes.

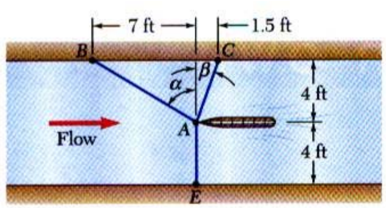
Law of Sines:

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\,\text{lb}}{\sin 58°}$$

$$T_{AB} = 3570\,\text{lb}$$

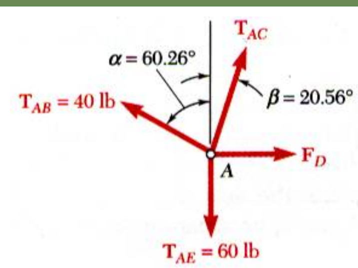
$$\boxed{T_{AC} = 144\,\text{lb}}$$

Sample Problem 2.6



It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

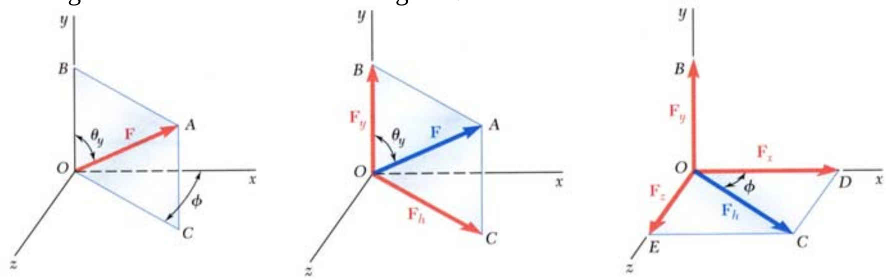
Determine the drag force exerted on the hull and the tension in cable *AC*.



$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$

Expressing a Vector in 3-D Space

If angles with some of the axes are given:



- The vector $\vec{F}$ is contained in the plane *OBAC*.
- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

$$F_h = F\sin\theta_y$$

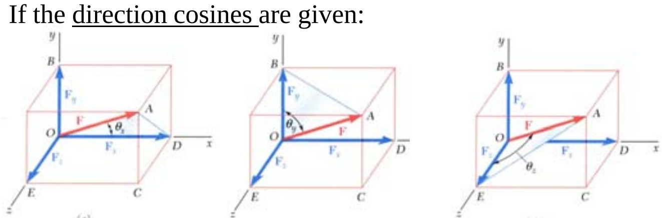
- Resolve F_h into rectangular components

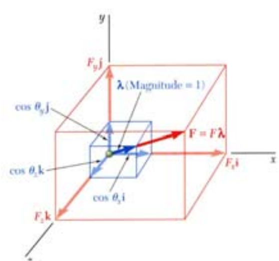
$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

$$F_y = F_h\sin\phi = F\sin\theta_y\sin\phi$$

Expressing a Vector in 3-D Space

If the direction cosines are given:





- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

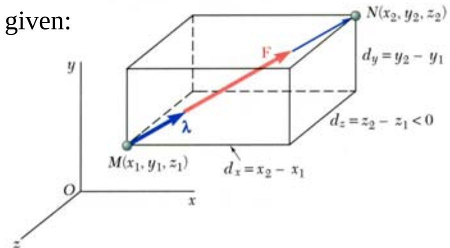
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

Expressing a Vector in 3-D Space

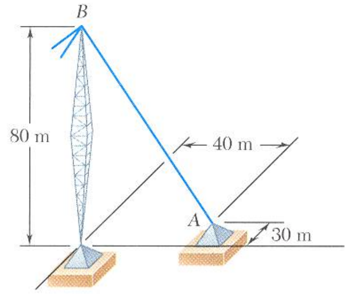
If two points on the line of action are given:

Direction of the force is defined by the location of two points,
$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = \text{vector joining } M \text{ and } N$$
$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$
$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$
$$\vec{F} = F\vec{\lambda}$$
$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$
$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Sample Problem 2.7



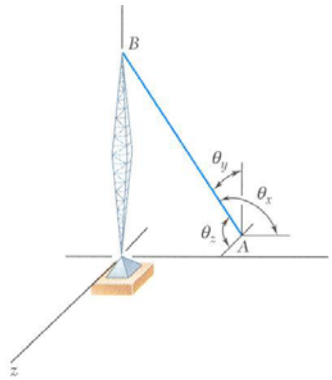
The tension in the guy wire is 2500 N. Determine:

a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force (the direction cosines)

$$\overrightarrow{AB} = (-40\text{m})\vec{i} + (80\text{m})\vec{j} + (30\text{m})\vec{k}$$
$$AB = \sqrt{(-40\text{m})^2 + (80\text{m})^2 + (30\text{m})^2}$$
$$= 94.3\text{ m}$$
$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$
$$\vec{F} = F\vec{\lambda}$$
$$= (2500\text{ N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$
$$= (-1060\text{N})\vec{i} + (2120\text{ N})\vec{j} + (795\text{ N})\vec{k}$$

Sample Problem 2.7

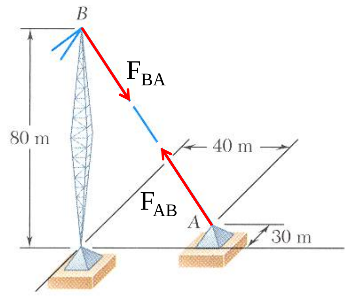


- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x\,\vec{i} + \cos\theta_y\,\vec{j} + \cos\theta_z\,\vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned}\theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ\end{aligned}}$$

What if…?



What are the components of the force in the wire at point B? Can you find it without doing any calculations?

SOLUTION:

- Since the force in the guy wire must be the same throughout its length, the force at B (and acting toward A) must be the same magnitude but opposite in direction to the force at A.

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$= (1060\,\text{N})\vec{i} + (-2120\text{ N})\vec{j} + (-795\text{ N})\vec{k}$$