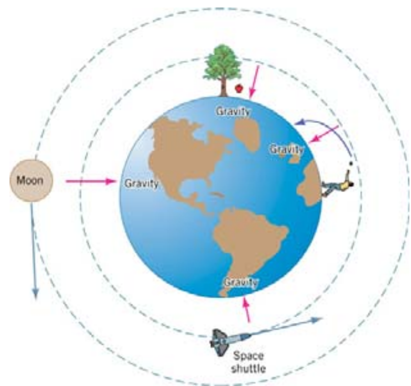
Chap 4 The Law of Universal Gravitation

Newton's Law of Universal Gravitation

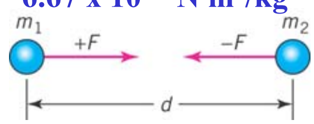
An attractive gravitational force exists between any two particles (with masses of m_1 and m_2 and with a separation of r between them) in the universe and its magnitude is

$$F = G\frac{m_1 \cdot m_2}{d^2}$$

This force is directed along the line joining the particles.

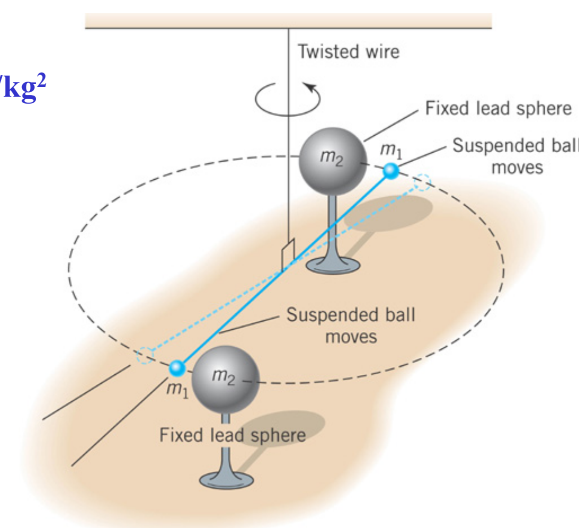


G = 6.67 x 10^{-11} N m^2/kg^2



Cavendish's Experiment

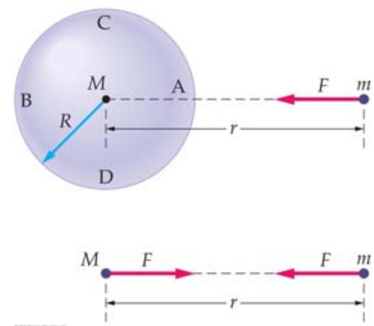
G = 6.67 x 10^{-11} N m^2/kg^2



Earth's Gravitational Force

The gravitational force that each uniform sphere of matter exerts on the other is the same as if each sphere were a particle with its mass concentrated at its center.

Any object on the surface of the earth is ~$6.38\text{x}10^6$m away from the center of the earth. The earth has a mass M of $5.98\text{x}10^{24}$ kg. The earth's gravitational force on an object of mass m near the surface of the earth is F = m (GM/R^2) = m ($9.8m/s^2$) = mg . That is why the gravitational acceleration g is $9.8m/s^2$ at sea level.



$$g = G\frac{M}{R_E^2} = \frac{6.67\times10^{-11}\cdot5.98\times10^{24}}{(6.38\times10^6)^2} = 9.8\ m/s^2$$

Weight and Mass

Weight is the gravitational force of an object (newton).

What is your weight on the Moon? Mass of the Moon is $7.3*10^{22}$ kg (0.0123 Earth), and the radius of the Moon is $1.7*10^6$ m (0.273 Earth).

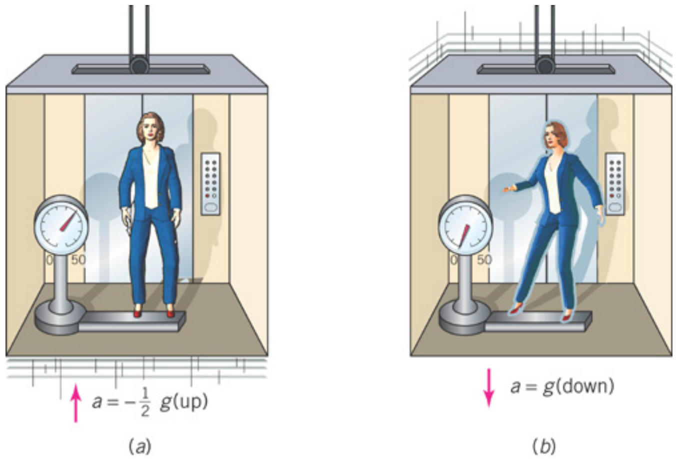
g~$1.62\ m/s^2$ on the Moon

3. What would you weigh if the Earth were four times as massive as it is and its radius were twice its present value?

4. How long would our year be if our Sun were half its present mass and the Earth's orbit was in the same place that it is now?

Apparent Weight

.. is the total force acting on a body, excluding the gravitational force.



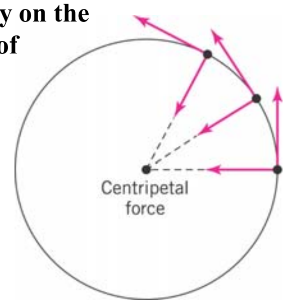
Gravity and Orbits

An object traveling with constant speed v on a circular orbit/path has a centripetal acceleration

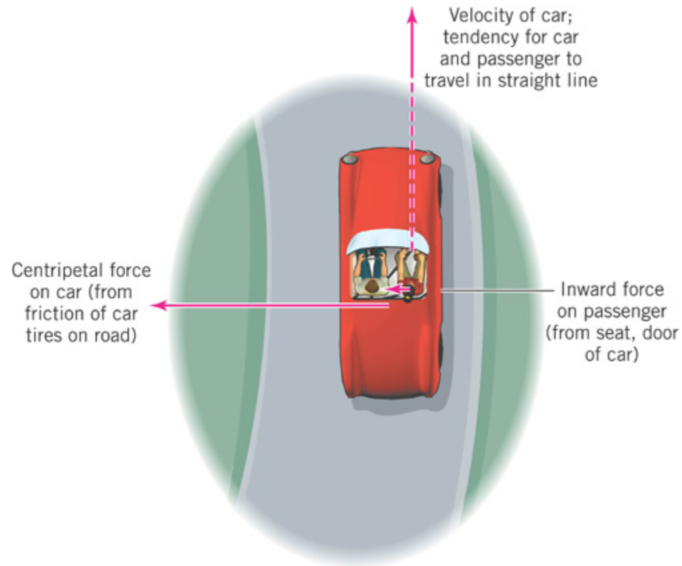
$$a_c = \frac{v^2}{r}$$

… which means that in order for the object to stay on the circular motion, it needs a net (centripetal) force of

$$F_c = ma_c = m\frac{v^2}{r}$$



Cars Turning



The Orbit Equation

A satellite orbiting the Earth or a planet orbit the Sun needs a net centripetal force, which is provided by the gravitational force between the two objects.

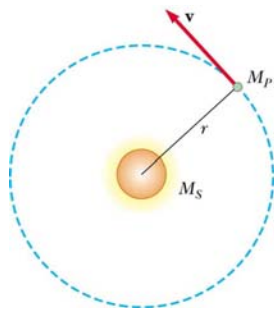
$$\frac{mv^2}{r} = G\frac{mM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

This is the explanation of Kepler's third Law!

Satellites in Circular Orbits

Kepler's Third Law: The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun.



$$\frac{t^2}{R^3} = const.$$

indep. of mass

$$t^2 = const \times R^3$$

Jupiter is 5.2 times farther from the Sun than the Earth is. How long is Jupiter's year?

Examples

11. Calculate the speed and period of a ball tied to a string of length 0.3 meters making 2.5 revolutions every second.

12. Calculate the average speed of the Moon in kilometers per second around the Earth. The Moon has a period of revolution of 27.3 days and an average distance from the Earth of 3.84×10^8 meters.

14. Calculate the centripetal force exerted on the Earth by the Sun. Assume that the period of revolution for the Earth is 365.25 days and the average distance is 1.5×10^8 km.

Geosynchronous Orbits

$$v = \frac{2\pi R}{t}$$

$$\frac{GM}{R} = \frac{4\pi^2 R^2}{t^2}$$

$$R^3 = \frac{GMt^2}{4\pi^2}$$

Desirable to have satellites circling the Earth with the period of exactly one day (24h). Where (how high) should be put the satellites?

R=42,000 km from center of Earth. (36,000 km above face of Earth).