Chapters 2 Kinematics

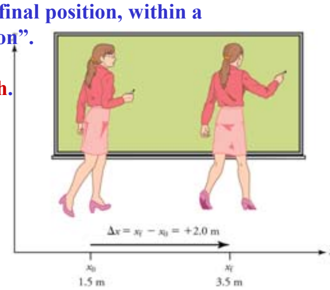
Position, Distance, Displacement

Mechanics: Kinematics and Dynamics.
Kinematics deals with motion, but is not concerned with the cause of motion.
Dynamics deals with the relationship between force and motion.

The word "displacement" implies the existence of an initial position (location) and a final position … often of the same object but at different times.

The displacement, $\Delta \mathbf{r}$, (as a result of an object's motion in a period of time) is the vector that points from the initial position toward the final position, within a suitable frame of reference. It is the "change in position".

The magnitude of a displacement has the unit of length.



1D Quantities: Implicit Vectors

A vector is characterized by a magnitude and a direction. (velocity, displacement, ...)

A scalar quantity has only a magnitude but no direction. (mass, temperature, ...)

If the motion of an object is limited along a straight line (its motion is one-dimensional), the displacement of this object can have only two directions (one being opposite to the other). If a coordinate system is chosen for the displacement (as in defining which direction is positive), the displacement may appear to be scalar-like. However, one should note that the "sign" of the displacement contains information on "direction".

average velocity

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

$$\Delta \mathrm{x} = \mathrm{x_f} - \mathrm{x_i}$$

Final position (coordinate)

Initial position (coordinate)

Average Velocity and Speed

average speed = distance traveled / elapsed time (scalar)

average velocity = displacement / elapsed time (vector)

Velocity and speed refer to the rate of change of an object's position (with respect to some stationary reference point, e.g. the origin).

The use of the terms "average velocity" and "average speed" implies a definite "range" (in time, or in space between two markers) in which the velocity or the speed (of an object) has been "averaged".

If an object's average velocity (/speed) is zero for a period of time, does its average speed (/velocity) have to be zero for that period of time?

No (/Yes)

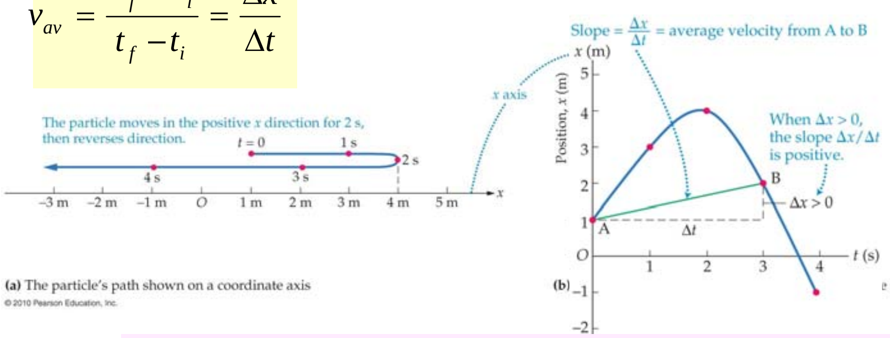
Avg. Velocity (1D) ⇔ Slope Of Line Through Two End Points

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta(y\ value)}{\Delta(x\ value)}$$

average velocity

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Note: slopes may have units



In the figure, what is the average velocity between 1 s and 3 s? between 0 s and 2s? between 1s and 4s?

Instantaneous Velocity (1D): Slope Of Tangent

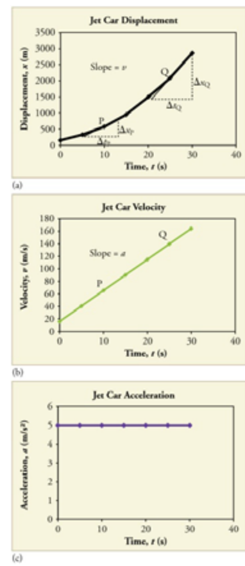
average velocity $\bar{v} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{\Delta x}{\Delta t}$

instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

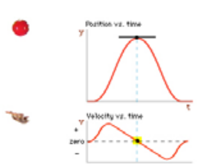
= slope of (the tangent line to) the x - t plot

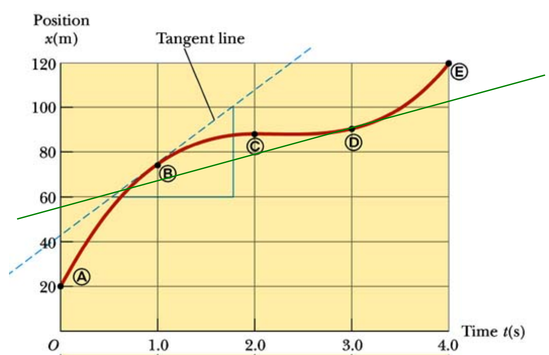
instantaneous speed $= |v|$



Instantaneous Velocity Exercises

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$





What is the instantaneous velocity at 1.0 s? **(100-60) m / (1.8 s – 0.5 s) = 31 m/s**

At which time is the instantaneous velocity the greatest: A, B, C or D?

Answer: A

What is the instantaneous velocity at 3.0 s? **~ 12 m/s**

Acceleration: the rate with which the velocity changes

Velocity is the rate of change of displacement (a vector).

Acceleration is the rate of change of velocity (also a vector).

The analysis of these two quantities is very similar.

The "rate of change" of a quantity is the "slope" of a plot of this quantity against time.

average acceleration (vector)

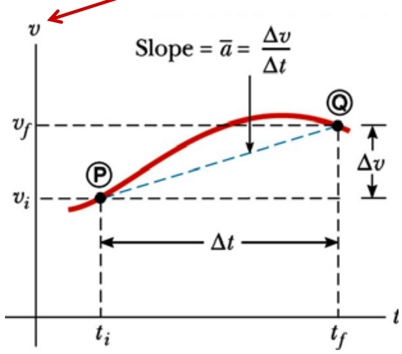
$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

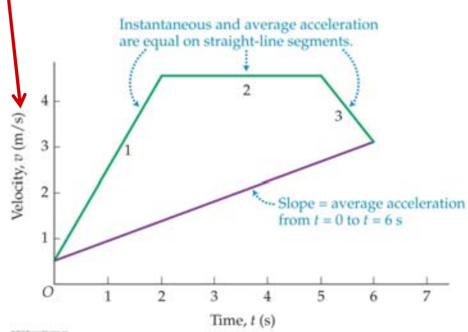
instantaneous acceleration (vector)

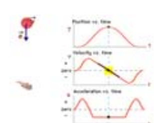
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Acceleration Examples

Note that the y-axis here is the "velocity" and not "distance" as before.

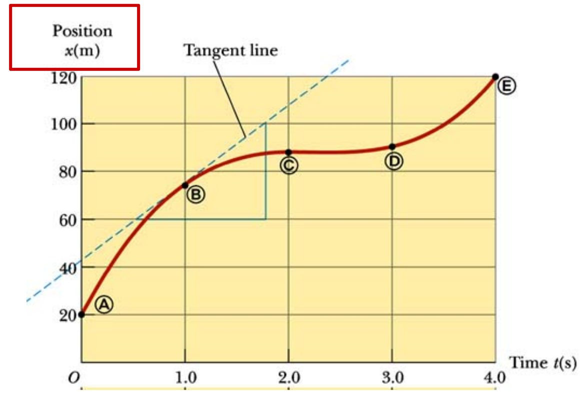






A "negative acceleration" does not necessarily imply that the "speed" of an object is reducing.

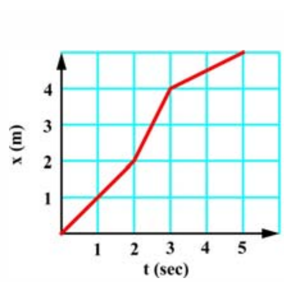
Judging Signs of Acceleration



Determination of the sign of acceleration from x-t plots.

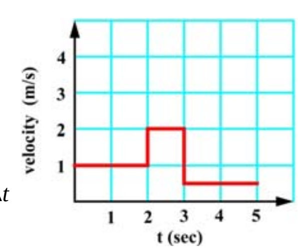
Accel. car

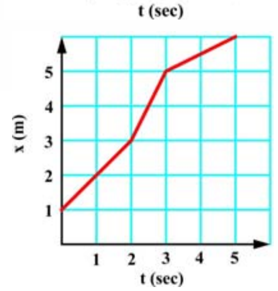
Displacement ⇔ Velocity



$$\bar{v} = \frac{x_f - x_i}{\Delta t}$$

$$x_f = x_i + \bar{v} \cdot \Delta t$$





The area under v versus t curve (from t_1 to t_2) is equal to the displacement between t_1 and t_2.

The relationship between a and v is the same as that between v and x.

Motion with Constant Acceleration

"Constant acceleration" means that **a** is constant (fixed).

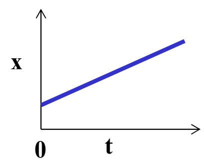
First, a trivial case …

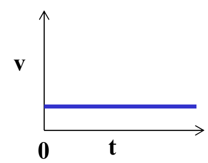
If the acceleration is zero, the velocity does not change with time, and the object will move with constant velocity, v.

If an object is at x_o at time t=0, it will be found at position

$$x = x_o + vt\,, \qquad \Delta x = x - x_o = vt$$

at time t = t.





Equations For Constant 1-D Acceleration

Non-zero constant acceleration: a (= constant)

Initial velocity at t = 0 : v_o

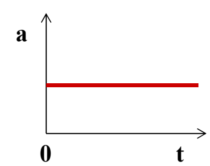
Velocity at time t is $v = v_o + at$

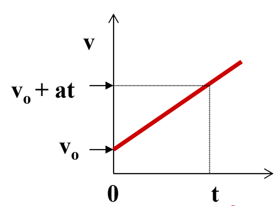
The average velocity between time zero and time t is

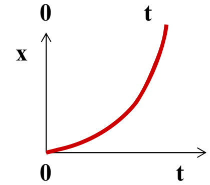
$$\bar{v} = \frac{1}{2}(v_o + v) = v_o + \frac{1}{2}at$$

The total displacement the object between time 0 and time t is the product of the average velocity and the time elapsed (t). If the object is assumed to be at the origin at t=0, its position at time t is

$$\Delta x = \bar{v}t = v_o t + \frac{1}{2}at^2$$

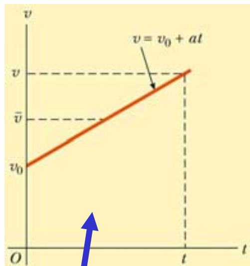






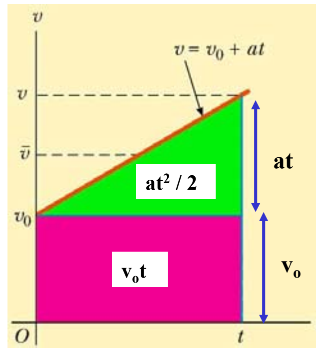
More on Displacement in 1D

$$\Delta x = \bar{v}\, t = v_o\, t + \frac{1}{2} a t^2$$



area of a trapezoid is
(1/2)*(top + base) * height

$= (1/2) * (v_o + v_o + at) * t$
$= v_o\, t + at^2 / 2$



area $= \Delta x = v_o\, t + at^2 / 2$

What if any of v_o, v_f, Δx or a were negative?

Equations For Constant 1-D Acceleration

$$\boxed{v = v_o + a t}$$ No Δx

$$\boxed{\Delta x = \frac{1}{2}(v_o + v)t}$$ No a

$$\boxed{\Delta x = v_o\, t + \frac{1}{2} a t^2}$$ No v

Add one more equation by multiplying the first two equations together

$$\boxed{v^2 = v_o^2 + 2 a \Delta x}$$ No t

To get the position of the object at time t, we need to add the displacement to the initial position of the object

$x = x_o + \Delta x$

Which Equation to Use?

2.6 Problem-Solving Basics for One-Dimensional Kinematics

- *The six basic problem solving steps for physics are:*

  Step 1. Examine the situation to determine which physical principles are involved.

  Step 2. Make a list of what is given or can be inferred from the problem as stated (identify the knowns).

  Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

  Step 4. Find an equation or set of equations that can help you solve the problem.

  Step 5. Substitute the knowns along with their units into the appropriate equation, and obtain numerical solutions complete with units.

  Step 6. Check the answer to see if it is reasonable: Does it make sense?

In the time it takes a car to accelerate along a straight line from 3m/s to 10m/s with a constant 2m/s² acceleration, how much distance will it have traveled?

A rock is dropped with an initial downward velocity of 1m/s from the top of a 50m building, what is its velocity just before it hits the ground? (the constant acceleration of any free falling object is 9.8m/s² downward)

A rock is dropped with an initial downward velocity of 1m/s from the top of a 50m building, how long does it take for it to reach the ground?

In 3 seconds, an object has accelerated from 0 to 20m/s with a constant acceleration. How much distance has it traveled during this time.

$$v = v_o + at$$

$$\Delta x = \frac{1}{2}(v_o + v)t$$

$$\Delta x = v_o t + \frac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

Not always necessary to use the most direct way. If you can see one way to get to the solution, take it.

Vertical Motion with Constant Gravitational Accel.

Acceleration due to gravity

a = g = 9.8m/s² downward

Or, g = - 9.8m/s² (because "up" is usually defined as the positive y direction.)
This acceleration is seen for all objects on earth, big or small, heavy or light, in the absence of air resistance.

How to work on problems such as "how high does it go?" or

Velocity vanishes at the highest point.

"how long is it in the air?"

Displacement as specified in the question is reached!

rock and feathers

$$v = v_o + gt$$

$$\Delta y = \frac{1}{2}(v_o + v)t$$

$$\Delta y = v_o t + \frac{1}{2}gt^2$$

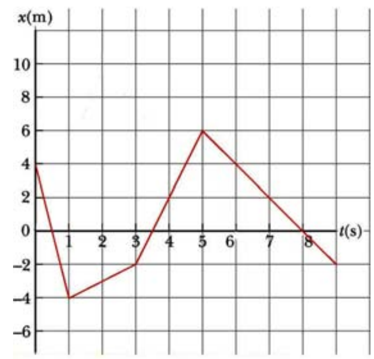
$$v^2 = v_o^2 + 2g\Delta y$$

Equation $At^2 + Bt + C = 0$ has solutions: $t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$ **Can usually tell which sign (+ or -) to use.**

Example Problems

57. A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

The position-time graph for an object moving along a straight path is shown. (a) Find the average velocity of this object during the time intervals 2.0 s to 4.0 s. (b) Find the instantaneous velocity at t=2.0 s. (c) Find the average acceleration between t= 2.0 s and t=4.0 s. (d) Find the average speed between t=0.0 s and t=8.0 s.



Review of Chapter 2

Displacement involves direction and distance (vector).

Average velocity = displacement/elapsed time

Instantaneous velocity = (average) velocity for an infinitesimally small time period.

Average acceleration = velocity variation / elapsed time

Instantaneous acceleration = velocity variation / infinitesimally small elapsed time

Equations for object undergoing constant acceleration in 1-D.

Free falling involves a constant acceleration of $9.8 m/s^2$ downward for all objects.