# Chapter 3: Two-Dimensional Kinematics

## Vectors: magnitude and direction

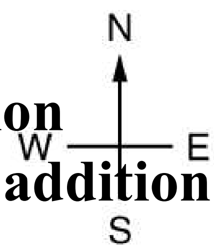
## Negative of a vector: reverse its direction

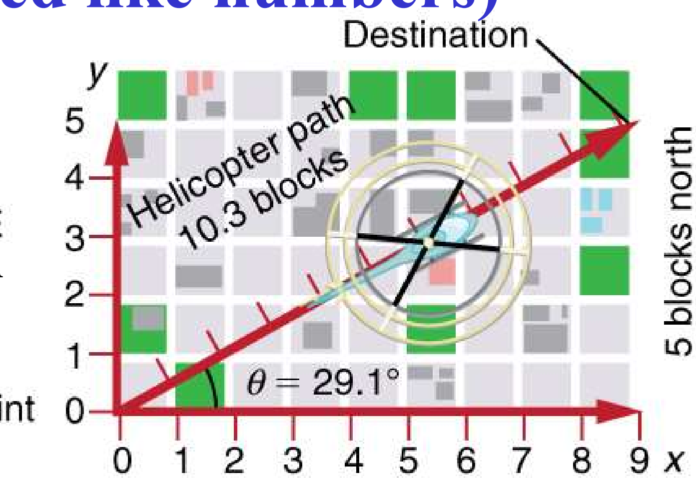
## Multiplying or dividing a vector by a scalar:

## Vectors in the same direction (treated like numbers)

## General Vector Addition:

**Triangle method of addition**
**Parallelogram method of addition**
**Components addition**




## Unit Vectors

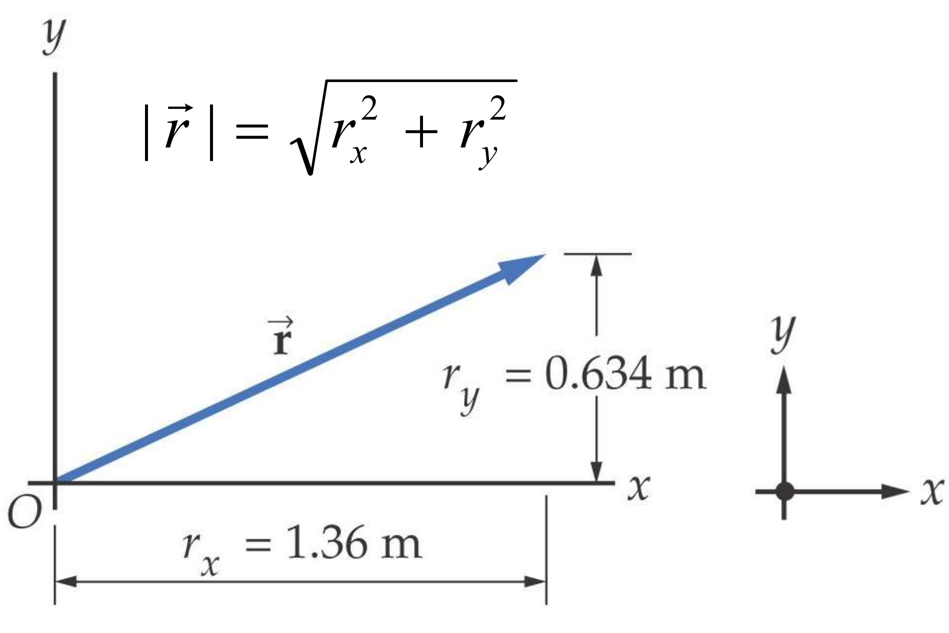
**Note that a vector has a magnitude and a direction, but it doesn't have a fixed "location" in space. It can be redrawn anywhere in space and still remains the same vector.**

# Coordinate Systems

**A coordinate system allows the locations (of objects) to be identified relative to the origin and a set of specified axes or directions. In two dimensions,**

**Cartesian coordinate system (x, y)**

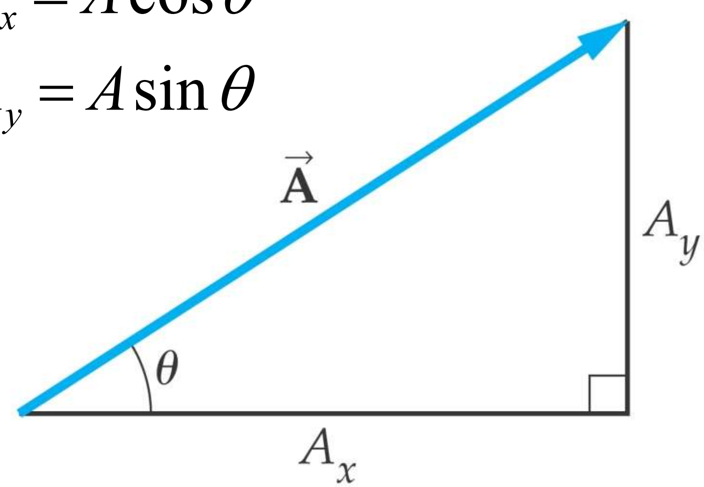
$$|\vec{r}\,| = \sqrt{r_x^2 + r_y^2}$$




**(b)**

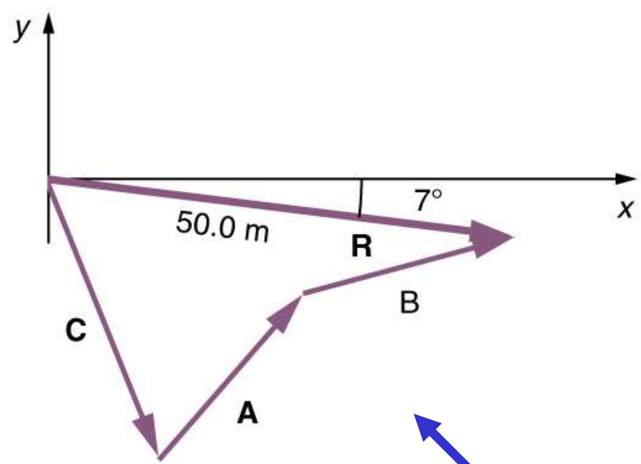
**Polar coordinates (r, θ)**

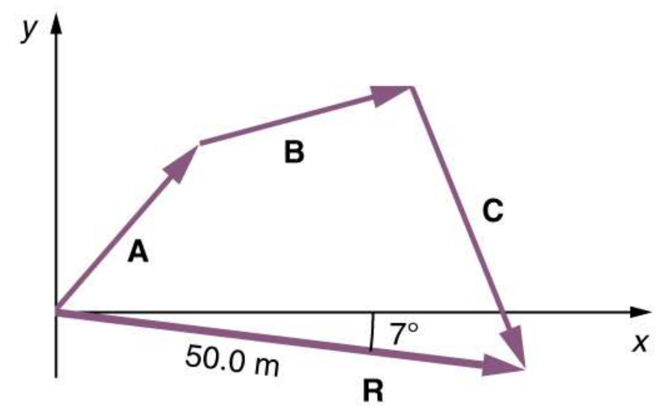
$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$



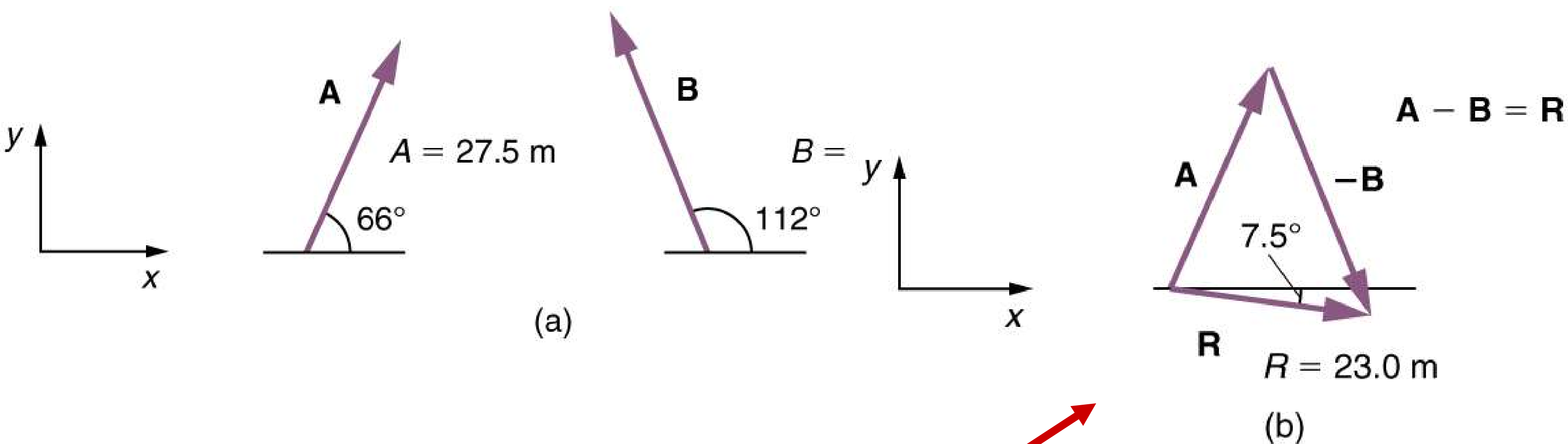

**polar coord.**

# Vector Addition




**vector addition**




**vector subtraction**

# Components of a Vector

**A two dimensional vector can be expressed as the sum of its components along two perpendicular axes.**

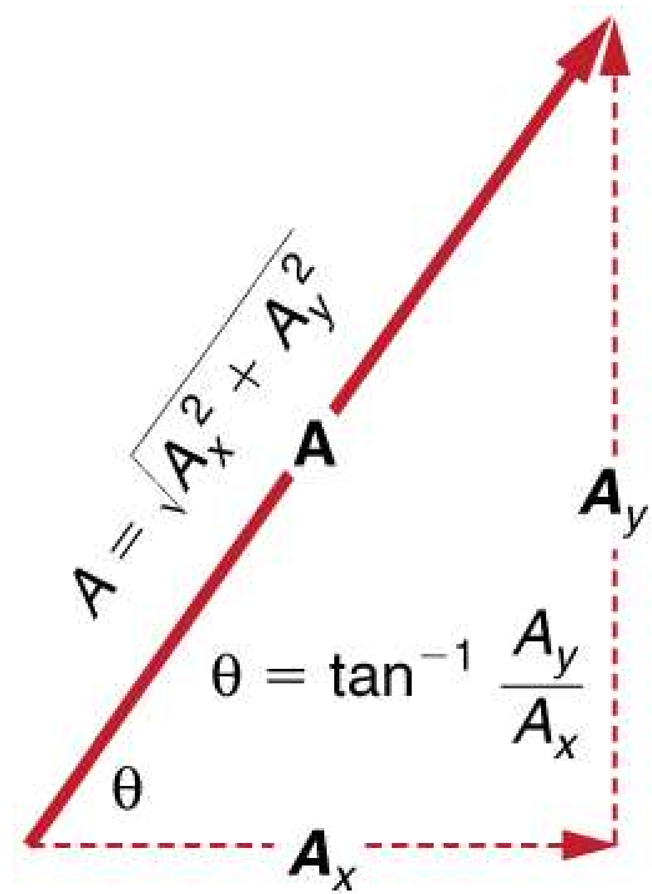
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

**$A_x = A \cos \theta$**

**$A_y = A \sin \theta$**

**$A = ( A_x^2 + A_y^2 )^{1/2}$**

$$\theta = \tan^{-1} \frac{A_y}{A_x}$$

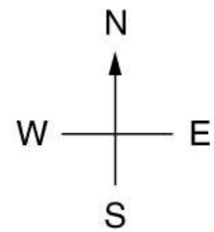


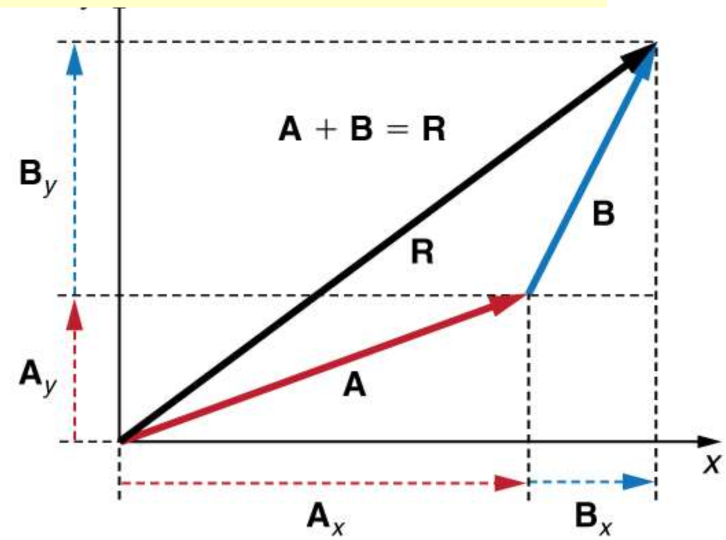
# Vector Addition by Components

$$\vec{A} = \vec{A}_x + \vec{A}_y \qquad\qquad \vec{B} = \vec{B}_x + \vec{B}_y$$

$$\vec{A} + \vec{B} = \vec{A}_x + \vec{A}_y + \vec{B}_x + \vec{B}_y$$

$$= \underbrace{(\vec{A}_x + \vec{B}_x)}_{\text{x-component}} + \underbrace{(\vec{A}_y + \vec{B}_y)}_{\text{y-component}}$$




$$|\vec{A} + \vec{B}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$\theta = \tan^{-1}\left(\frac{A_y + B_y}{A_x + B_x}\right)$$

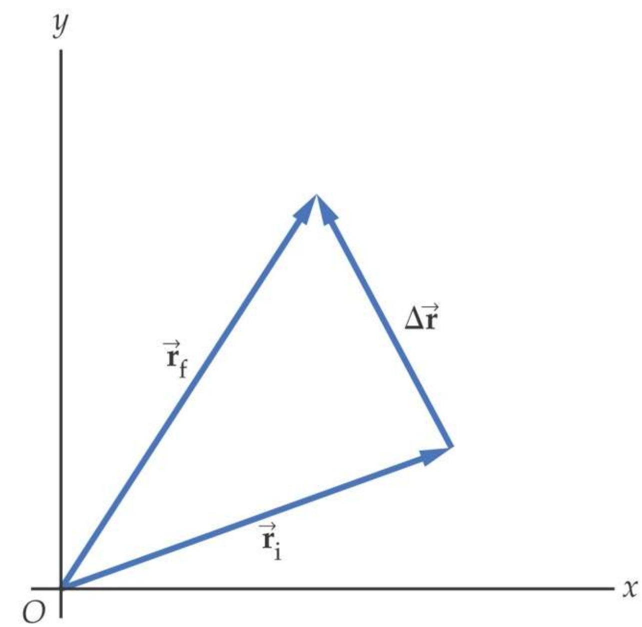
1. **Find the components of each vector to be added.**
2. **Add the *x*- and *y*-components separately.**
3. **Find the resultant vector.**

# Displacement, Vel., and Accel. in 2D

**displacement** $$\overrightarrow{\Delta r} \equiv \vec{r}_f - \vec{r}_i$$

**average velocity** $$\bar{\vec{v}} \equiv \frac{\overrightarrow{\Delta r}}{\Delta t}$$

**instantaneous velocity** $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t}$$




**average acceleration** $$\bar{\vec{a}} \equiv \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

**instantaneous acceleration** $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t}$$

# Position, Velocity, and Acceleration: 1D to 2D or 3D

**For 1-D motion, we used to know that**

**….. the slope at a specific time of a plot of the position as a function of time is the instantaneous velocity at that specific time;**

**… the slope at a specific time of a plot of the velocity as a function of time is the instantaneous acceleration at that specific time, etc.**

**Are these still correct in 2D?**

$$\vec{v}_{avg} \equiv \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t} \qquad \vec{r}_i = \vec{x}_i + \vec{y}_i \qquad \vec{r}_f = \vec{x}_f + \vec{y}_f$$

$$\vec{v}_{avg} = \frac{\vec{x}_f + \vec{y}_f - (\vec{x}_i + \vec{y}_i)}{\Delta t} = \frac{(\vec{x}_f - \vec{x}_i) + (\vec{y}_f - \vec{y}_i)}{\Delta t} = \frac{\overrightarrow{\Delta x} + \overrightarrow{\Delta y}}{\Delta t}$$

$$\bar{v}_x = \frac{\overrightarrow{\Delta x}}{\Delta t} \qquad \bar{v}_y = \frac{\overrightarrow{\Delta y}}{\Delta t}$$

# Position, Velocity, and Acceleration: 1D to 2D or 3D

**For 1-D motion, we used to know that**

**….. the slope at a specific time of a plot of the position as a function of time is the instantaneous velocity at that specific time;**
**… the slope at a specific time of a plot of the velocity as a function of time is the instantaneous acceleration at that specific time, etc.**

**Are these still correct in 2D?**

**They are still valid for motions in 2D or 3D, as far as the components (of position, velocity, acceleration) along any specific direction are concerned.**

**For example:**
**The slope of the y-component of the velocity of an object, plotted against time, is the y-component of the instantaneous acceleration.**
**The slope of x-component of the displacement of an object, plotted against time, is the x-component of the instantaneous velocity, etc.**

# Motion in Two Dimensions

**The x-component of the translational (non-rotational) motion of an object is independent of its motion in the y-direction.**

**Motion in the *x*- and *y*-directions should be solved separately:**

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \frac{1}{2}(v_{x0} + v_x)t$$

$$\Delta x = v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2 a_x \Delta x$$

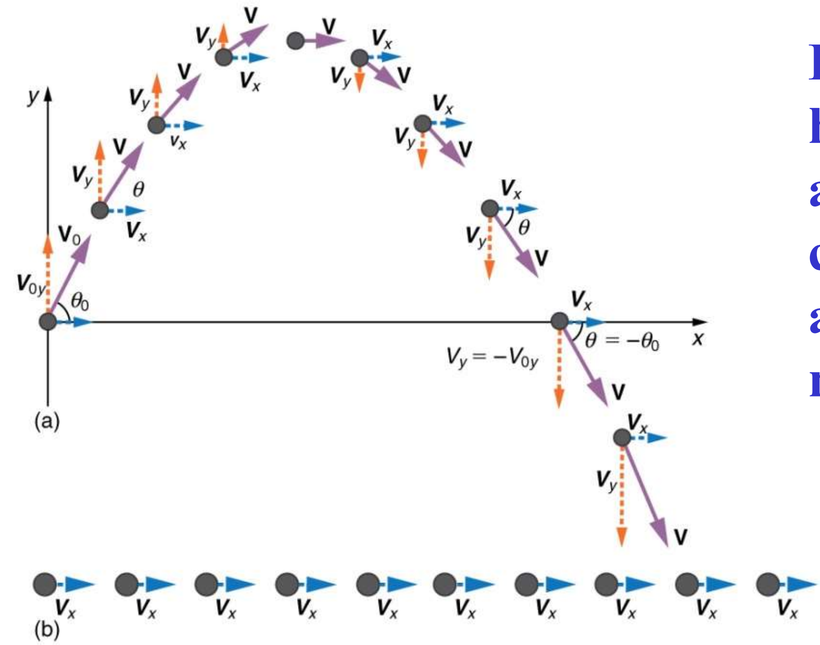
$$v_y = v_{y0} + a_y t$$

$$\Delta y = \frac{1}{2}(v_{y0} + v_y)t$$

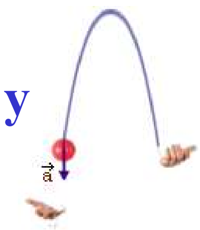
$$\Delta y = v_{y0} t + \frac{1}{2} a_y t^2$$

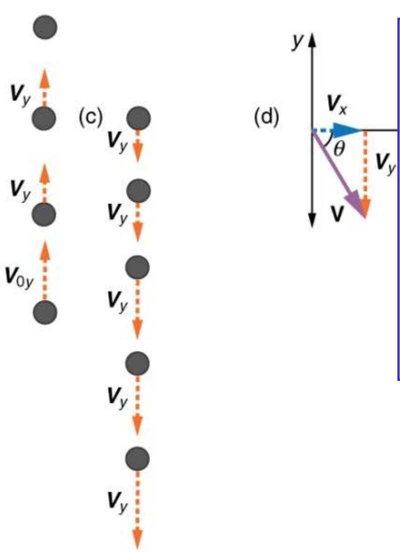
$$v_y^2 = v_{y0}^2 + 2 a_y \Delta y$$

# Projectile Motion




**In the absence of air resistance, the horizontal or x component of the acceleration is zero, and the vertical or y component of the acceleration is the acceleration due to gravity. These two motions are independent of each other.**

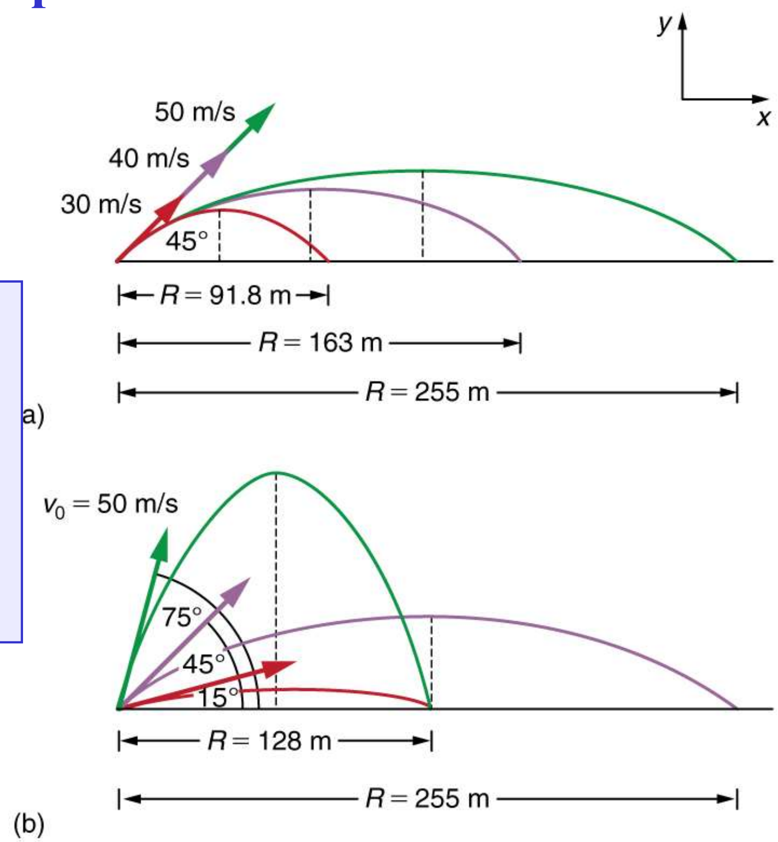



$$\Delta x = v_{x0}\, t$$

$$\Delta y = v_{y0}\, t + \frac{1}{2} g\, t^2$$

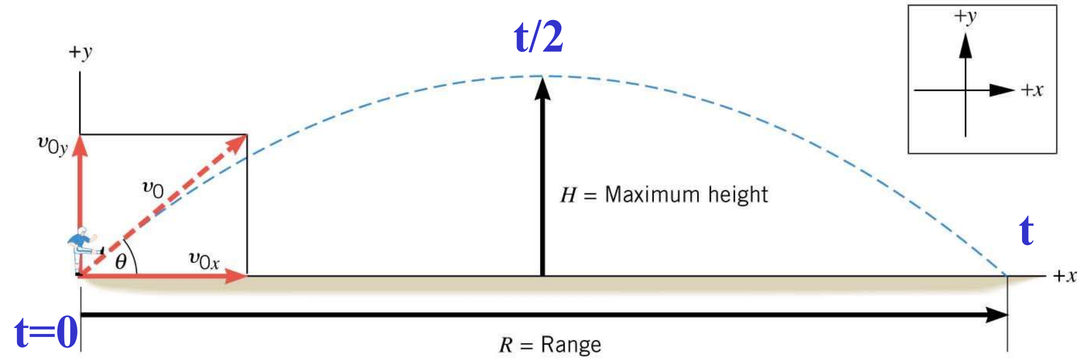
**-9.8 m/s²**



# General Launch Angle

$$\Delta x = v_{ox}\, t$$

$$\Delta y = v_{y0}\, t - (4.9 m/s^2) t^2$$




**How much time is it in the air?**

$$t = 2 v_{y0} / g = 2 v_o \sin\theta / g$$

**How high in air does it go?**

$$H = \frac{v_{y0}}{2}(t/2) = \frac{v_o^2 \sin^2\theta}{2g}$$

**How far does it go?**

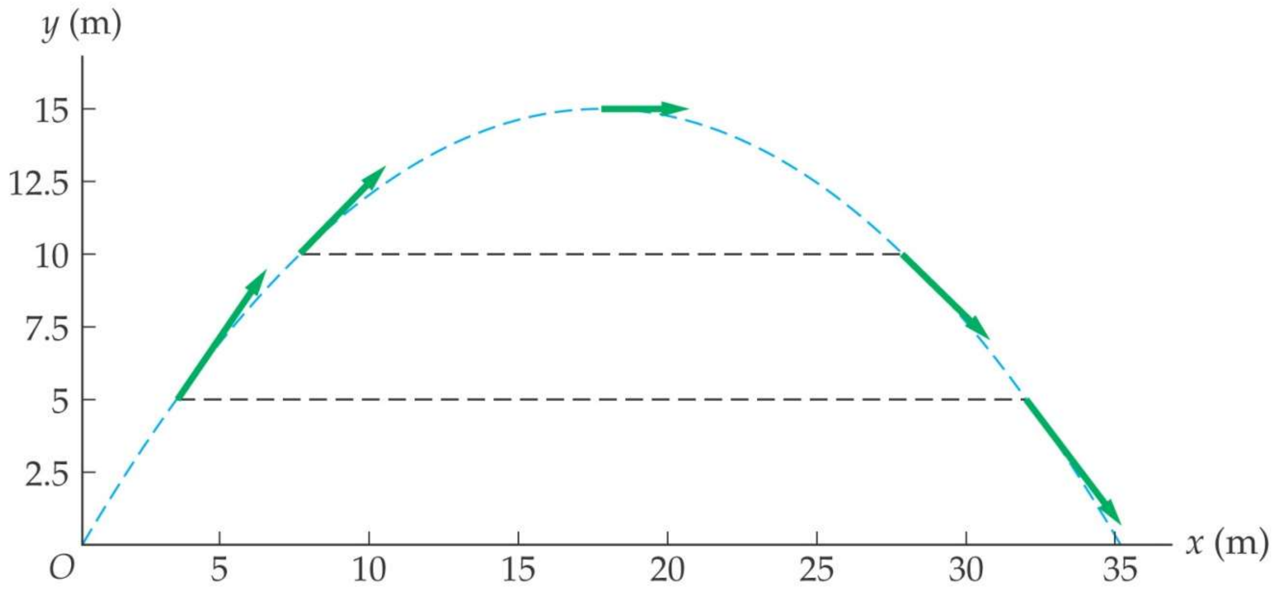
$$R = v_{x0}\, t = 2 v_o^2 \sin\theta \cos\theta / g = v_o^2 \sin 2\theta / g$$

**What's the farthest a ball will go when thrown with the same $v_o$?**

**Range**

# Projectile Motion: Key Characteristics

**Symmetry in projectile motion:**

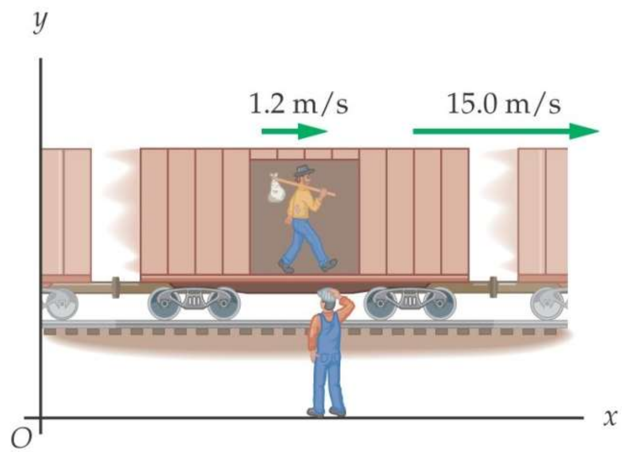


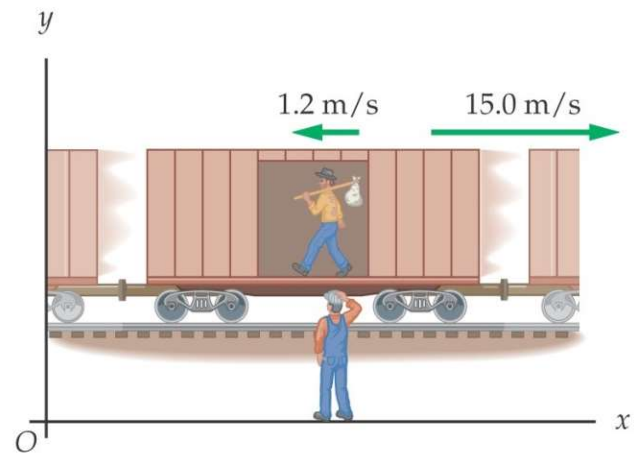

**Note that the projectile trajectory intercept with a certain height (below maximum height) at two places (two x's). The speeds of the projectile at the same height are identical.**

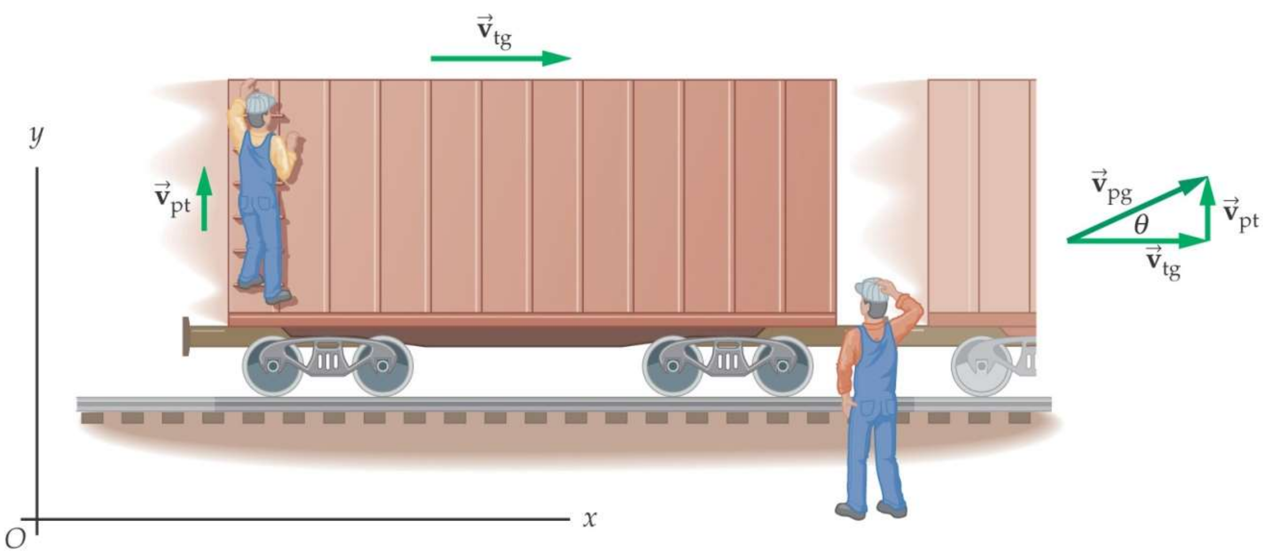
**Note also that there are two specific times that the projectile is at a certain height (below maximum). This is the reason that the quadratic equation for time has two "roots".**

# Relative Motion

**What is the velocity of the person on the moving train as viewed from the ground?**




$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$$

# Relative Velocity

The velocity of A relative to B, $V_{AB}$ , is the velocity of A as viewed from the vantage point of B.

Naturally, $$\vec{v}_{AB} = -\vec{v}_{BA}$$

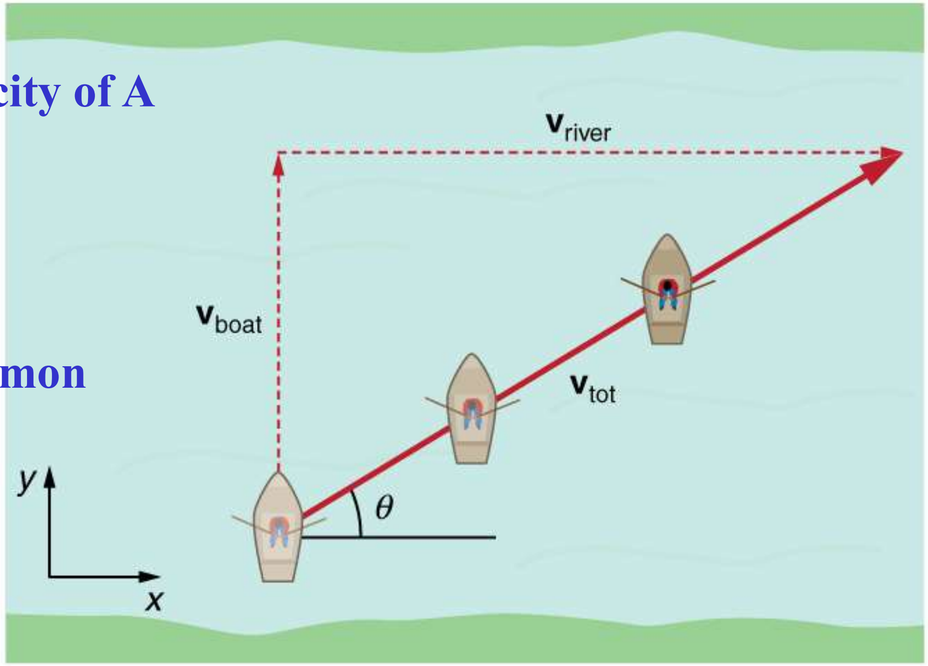
If the velocities of A and B are known to a common observer, C, as $V_{AC}$ and $V_{BC}$, respectively,

then, $$\vec{v}_{AB} = \vec{v}_{AC} - \vec{v}_{BC}$$

which can be written as

$$\boxed{\vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}}$$

A, B, C, can be any three objects.




What is velocity of the boat relative to land?

$$\vec{v}_{BoatShore} = \vec{v}_{BoatWater} + \vec{v}_{WaterShore}$$

Mnemonic device: Think of A, B, C as three points on a piece of paper and $V_{AB}$ as the vector from A to B, and vice versa.

# Chapter 3 Example Problems

**34. An arrow is shot from a height of 1.5 m toward a cliff of height H. It is shot with a velocity of 30 m/s at an angle of 60$^o$ above the horizontal. It lands on the top edge of the cliff 4.0 s later. (a) What is the height of the cliff? (b) What is the maximum height reached by the arrow along its trajectory? (3) What is the arrow's impact speed just before hitting the cliff?**

# Chapter 3 Example Problems

**37. Serving at a speed of 170 km/h, a tennis player hits the ball at a height of 2.5 m and an angle $\theta$ below the horizontal. The service line is 11.9 m from the net, which is 0.91 m high. What is the angle $\theta$ such that the ball just crosses the net? Will the ball land in the service box, whose out line is 6.40 m from the net?**

# Chapter 3 Example Problems

**60. (a) An airplane is flying in a jet stream that is blowing at 45.0 m/s in a direction 20$^o$ south of east. Its direction of motion relative to the Earth is 45.0$^o$ south of west, while its direction of travel relative to the air is 5.00$^o$ south of west. What is the airplane's speed relative to the air mass? (b) What is the airplane's speed relative to the Earth?**

# Chapter 3 Example Problems

**An arrow is shot at an angle of 75.0$^o$ above the horizontal from the ground level toward a cliff. It lands with a descending angle of 10.0$^o$ on the cliff 3.00 s later. What is the height of the cliff? Neglect air resistance. (Hint: Write the constant horizontal velocity as Vx. Express the y-component of the initial and final velocities with Vx and the angles. Relate the y-component of the two velocities to find Vx.)**

**An arrow is shot with an initial speed of 30.0 m/s from the ground level toward a cliff of 35.0 m height. It lands with a descending angle of 15.0$^o$ on the cliff a short time later. What is the total flight time of the arrow? Neglect air resistance. (Hint: first figure out the landing speed on the cliff. Then use the y-component of the final velocity to figure out the time it took.)**

**An arrow is shot with an initial speed of 30.0 m/s from the ground level toward a very tall vertical cliff 35.0 m (horizontal distance) away. It hits the side of the cliff horizontally. What is a possible total flight time of the arrow? Neglect air resistance. (Hint: If the cliff weren't there, the arrow would have landed with a range of 70.0 m on the ground. There are two possible times of flight!)**

# Summary of Chapter 3

• **Scalar: number, with appropriate units**

• **Vector: quantity with magnitude and direction**

• **Vector components:** $A_x = A \cos \theta$, $B_y = B \sin \theta$

• **Magnitude:** $A = (A_x^2 + A_y^2)^{1/2}$

• **Direction:** $\theta = \tan^{-1} (A_y / A_x)$

• **Graphical vector addition: Place tail of second at head of first; sum points from tail of first to head of last**

•**Relative motion:** $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$

•**Projectile motion: Horizontal velocity component is constant. Vertical velocity component increases at a rate of g. The position of an object in motion is described by both the horizontal and the vertical coordinates.**