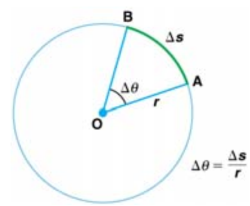
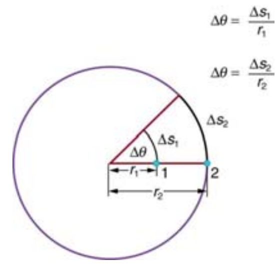
Chapter 6 Uniform Circular Motion and Gravitation





angle θ and angular displacement Δθ

average angular velocity $$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Is ω a scalar or a vector?

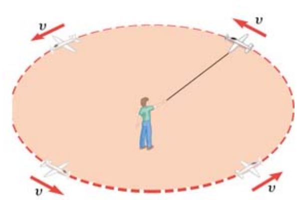
instantaneous angular velocity $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

Chapter 6 Uniform Circular Motion and Gravitation

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

The speed, v, is related to the radius of the circle, r, and the period, T, of the circular motion

$$v = \frac{2\pi r}{T}$$



The "angular" speed, ω, is the rate of change of the angle of the object in circular motion. The angle is expressed in radians rather than degrees.

$$\omega = \frac{v}{r} = \frac{2\pi}{T}$$

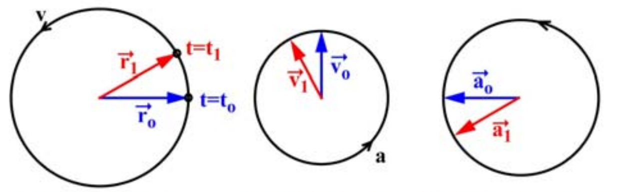
What is the motion of the object on a string, if the string suddenly snaps?

What is the acceleration of an object in uniform circular motion?

Centripetal Acceleration

radius / speed

$v = 2\pi r / T$ $\qquad$ $a = 2\pi v / T$



Position Plot $\qquad$ Velocity Plot $\qquad$ Acceleration Plot

$$\boxed{a = 2\pi v / T = v^2 / r = v\omega = r\omega^2}$$

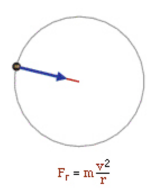
The centripetal acceleration of an object moving with a speed of v on a circular path of radius r has a magnitude of v^2/r and a direction which points toward the center of the circle and continually changes direction as the object moves.

Centripetal Force

All accelerations are due to non-zero net forces. (F=ma) An object with a mass m moving at a speed v on a circular path of radius r has a centripetal acceleration of v^2/r. Therefore, there must be a centripetal force of

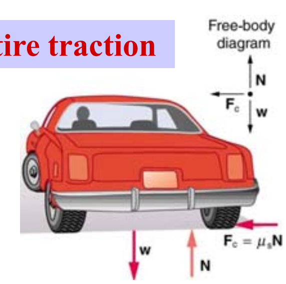
$$F_c = mv^2 / r$$

acting on it. For an object on a string, this force is provided by the tension of the string. And for moon circling the earth, the gravitational force; for cars cornering without skidding, the (static) frictional force between the tires and the pavement. An object traveling with a speed of v needs to receive such a force in order to follow a circular orbit.



Cent. Force

tire traction



condition for no-skid turning

$$mg\mu_S \geq \frac{mv^2}{r} \qquad \mu_S \geq \frac{v^2}{rg}$$

Banked Curves

In the absence of friction, a car can still negotiate a turn if the road is banked.

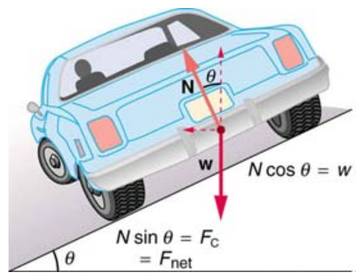
$$F_C = n \sin\theta = \frac{mv^2}{r}$$

$$n \cos\theta = mg$$

$$\frac{n\sin\theta}{n\cos\theta} = \frac{mv^2/r}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

frictionless bank



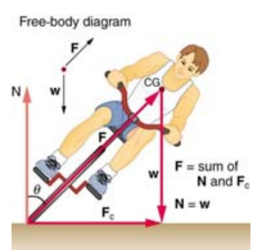
Examples

8. When kicking a football, the kicker rotates his leg about the hip joint. (a) If the velocity of the tip of the kicker's shoe is 35.0 m/s and the hip oint is 1.05 m from the tip of the shoe, what is the shoe tip's angular velocity? (b) The shoe is in contact with the initially stationary 0.500 kg football for 20.0 ms. What average force is exerted on the football to give it a velocity of 20.0 m/s? (c) Find the maximum range of the football, neglecting air resistance.

28. Part of riding a bicycle involves leaning at the correct angle when making a turn. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components – friction parallel to the orad and the vertical normal force. (a) Show that θ is related to the speed v and the radius of curvature r of the turn as

$$\theta = \tan^{-1} v^2/rg$$

(b) Calculate θ for a 12.0 m/s turn of radius 30.0 m.

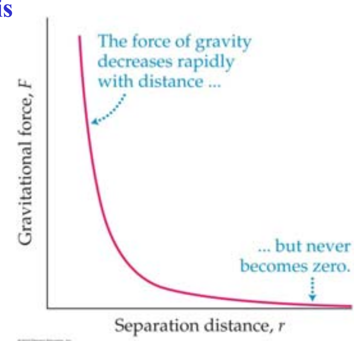


Gravity

Newton's Law of Universal Gravitation

An attractive gravitational force exists between any two particles (with masses of m_1 and m_2 and with a separation of r between them) in the universe and its magnitude is

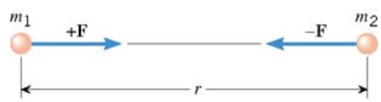
$$F = -G\frac{m_1 m_2}{r^2}$$



This force is directed along the line joining the particles. Being attractive, this force is sometimes written with a minus sign.

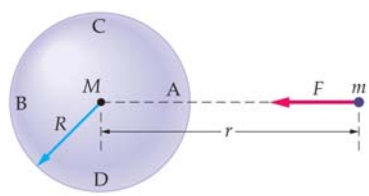
$G = 6.67 \times 10^{-11}$ N m^2/kg^2

In comparison, electrostatic forces follow a similar "inverse square" law with a force constant of 3.6×10^{10} N m^2/Coulomb^{-2}



Earth's Gravitational Force

The gravitational force that each uniform sphere of matter exerts on the other is the same as if each sphere were a particle with its mass concentrated at its center. (Caveat: This is only true for objects that are completely external to each other)



Any object on the surface of the earth is ~6.38×10^6m away from the center of the earth. The earth has a mass M of 5.98×10^{24} kg. The earth's gravitational force on an object of mass m near the surface of the earth is F = m (GM/R^2) = m (9.8m/s^2) = mg . That is why the gravitational acceleration g is 9.8m/s^2 at sea level.

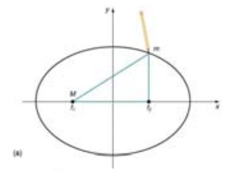
$$g = G\frac{M}{R_E^2} = \frac{6.67\times 10^{-11}\cdot 5.98\times 10^{24}}{(6.38\times 10^6)^2} = 9.8\ m/s^2$$

Exercise 12-2: What is g on the Moon? Mass of the Moon is 7.3×10^{22} kg (0.0123 Earth), and the radius of the Moon is 1.7×10^6 m (0.273 Earth).

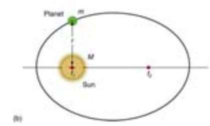
~1.62 m/s^2

Kepler's Laws of Orbital Motion

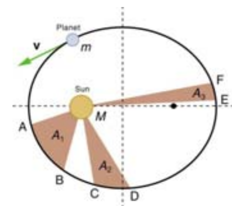
Kepler's First Law: An object bound by gravitational force to another moves in an elliptical orbit, with the center of mass (Sun) occupying one of the foci.



Kepler's Second Law: A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.



Elliptical orbit **Kepler's 2nd Law**



What conservation law led to Kepler's Second Law?

Satellites in Circular Orbits

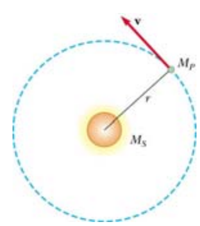
$$F_C = G\frac{M_P M_S}{r^2} = \frac{M_P v^2}{r}$$

$$v = \sqrt{\frac{G M_S}{r}}$$

T: period $$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM_S}}$$

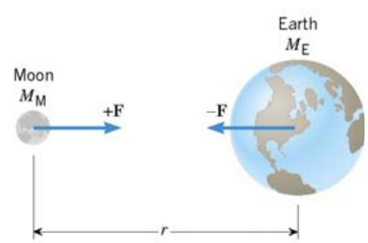
indep. of mass

$$\boxed{T^2 = \frac{4\pi^2 r^3}{GM_S} = K_S\, r^3}$$



Kepler's Third Law: The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun.

Why is the momentum of the orbiting planet not conserved?



Example Problem

43. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are usful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). Calculate the radius of such an orbit based on the data for the moon.
(Average orbital radius = 3.84×10^5 km; Period = 0.07481 y)

Summary of Chapter 6

- **Angle, angular velocity**

$$\Delta\theta = \frac{\Delta s}{r} \qquad \omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

- **Centripetal Acceleration**

$$a_C = \frac{v^2}{r} = r\omega^2$$

- **Centripetal force:**

$$F_C = m\frac{v^2}{r} = mr\omega^2$$

Summary of Chapter 6

- **Force of gravity between two point masses:**

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2} \qquad g = \frac{GM_E}{R_E^2}$$

Kepler's laws:

1. **Planetary orbits are ellipses, Sun at one focus**
2. **Planets sweep out equal area in equal time**
3. **Square of orbital period is proportional to cube of distance from Sun**

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2} = (\text{constant}) r^{3/2}$$

- **Escape speed:** $v_e = \sqrt{\dfrac{2GM_E}{R_E}}$