

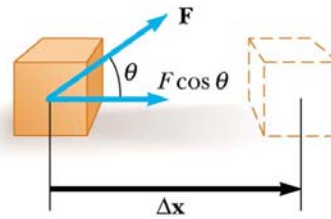
Chapter 7: Work, Energy and Resources

The **work** done on an object by a constant force F is

$$W = (F \cos \theta) \Delta x,$$

Where F is the magnitude of the force, Δx is the magnitude of the displacement, and θ is the angle between the force and the displacement.

Work does not depend on velocity or time. It can be positive or negative (when $\cos \theta < 0$).



If displacement is in the direction of the force, $\theta=0$,

$$W = F \Delta x.$$

SI Unit of Work:

$$\text{newton} \times \text{meter} = \text{joule (J)}$$

What if two or more (constant) forces act on an object at the same time and cause a displacement?

The same displacement is used to calculate separately the work done by each force.

More About Work

Work can be thought of as the **PRODUCT** of

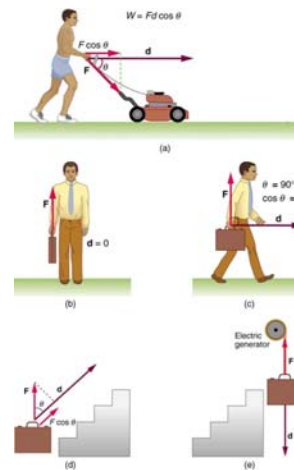
- (1) Component of force in the direction of displacement AND the displacement
- (2) Component of displacement in the direction of force AND the force

$$W = \vec{F} \cdot \Delta \vec{r} = F_x \Delta x + F_y \Delta y$$

Work can be positive, negative or zero.

Work may depend on path (non-conservative force).

Total work is sum of work done by different forces.



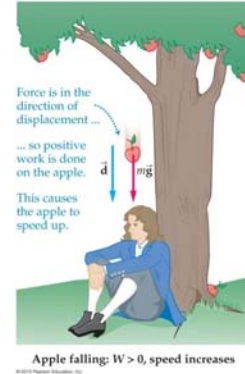
What Changes When Work Is Done?

Without external forces, Newton's first law tells us that the velocity of an object does not change. When an external force F is applied to the object, an acceleration $a = F/m$ takes place. If the work done on the object is positive, as shown here, the velocity of the object increases. If negative work is done, the velocity of the object decreases. By how much does the velocity increase or decrease? Is this increase or decrease in velocity related to the work done?

$$v_f^2 - v_o^2 = 2 a \Delta x = 2 \frac{F \Delta x}{m} = \frac{2W}{m}$$

W = work done

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2 = W$$



Kinetic Energy

The kinetic energy KE of an object with mass m and speed v is given by

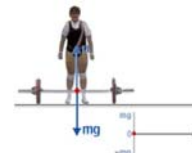
$$KE = \frac{1}{2} m v^2$$

SI Unit of Work: joule (J)

The Work-Kinetic Energy Theorem

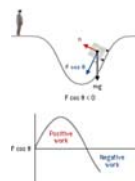
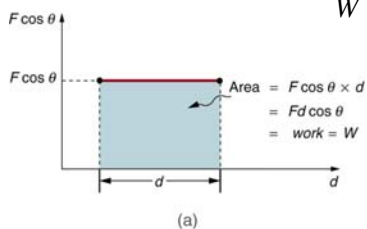
When a net external force does work W on an object, the kinetic energy of the object changes from its initial value of KE_i to a final value of KE_f , the difference between the two values being equal to the work:

$$W = KE_f - KE_i = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_o^2$$

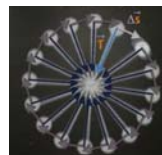
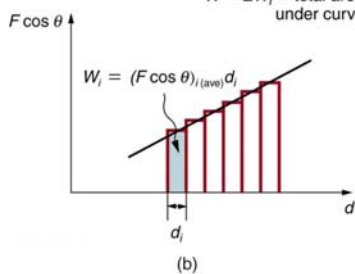


Work Done by A Variable Force

$$W = (F \cos \theta)_1 \Delta s_1 + (F \cos \theta)_2 \Delta s_2 + \dots$$



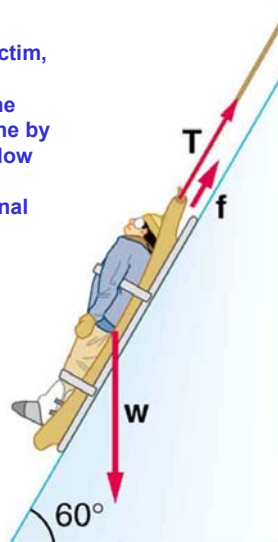
$$W = \sum W_i = \text{total area under curve}$$



Example Problems

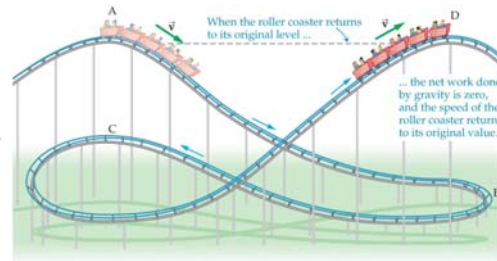
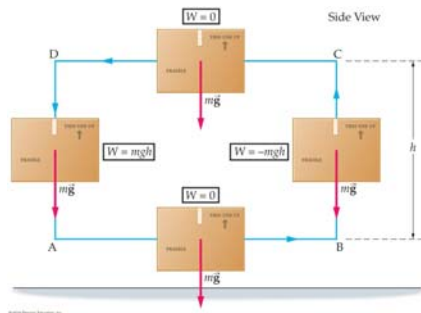
8. Suppose the ski patrol lowers a rescue sled and victim, having a total mass of 90.0 kg, down a 60.0° slope at constant speed. The coefficient of friction between the sled and the snow is 0.100. (a) How much work is done by friction as the sled moves 30.0 m along the hill? (b) How much work is done by the rope on the sled in this distance? (c) What is the work done by the gravitational force on the sled? (d) What is the total work done?

13. A car's bumper is designed to withstand a 1.1-m/s collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.



Potential Energy and Conservation of Energy

Conservative Force: is one that does zero total work on any closed path.



Force is conservative, if the work done by a force in going from an arbitrary point A to an arbitrary point B is independent of the path from A to B.

Potential Energy

The work done by a conservative force can be thought of as the release of stored energy U (potential energy). The amount of stored potential energy changes with position.

$$W = U_i - U_f = -\Delta U$$

POTENTIAL ENERGY at point A

... is the **negative** of the work done by a conservative force from point 0 to point A.

... is the work that need to be **done against** a conservative force to bring an object at rest from point 0 to point A.

... is the **energy stored** in a conservative force field at point A

... is the **potential (ability, tendency,..)** of a conservative force field to do work, starting from point A

Gravitational Potential Energy

$$W_{\text{gravity}} = mg (y_i - y_f)$$

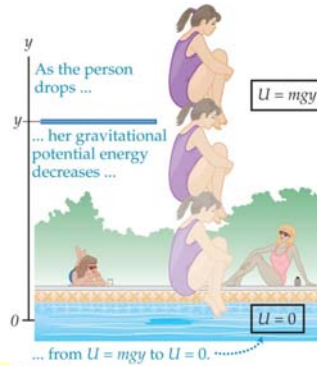
$$W = U_i - U_f = -\Delta U$$

The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height y of the object relative to an arbitrary zero level:

$$PE = mgy$$

$$W_g = PE_i - PE_f$$

SI Unit of PE is joule (J).



The choice for the location of “zero potential energy” is arbitrary, because only change in potential energy has physical consequence.

Work done against
Gravitation

Potential Energy of Compressed (of Stretched) Spring

How much work is required to compress a spring by a displacement of x from its unstrained position? We know that

$$W = F s$$

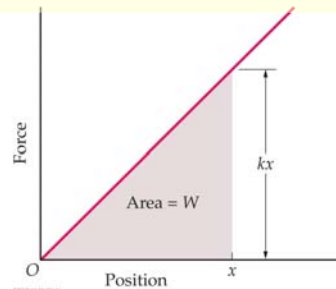
for a constant force. For a variable force, the total work is the sum of works done in all segments of the displacement

$$W = \sum F \Delta s$$

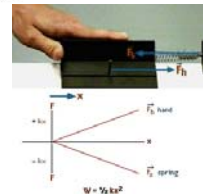
As the diagram shows, the work done is $kx^2/2$ to compress (or dilate) the spring by x.

Therefore, the elastic potential energy of a stretched or compressed spring is

$$PE_{\text{elastic}} = kx^2/2$$



Spring-block



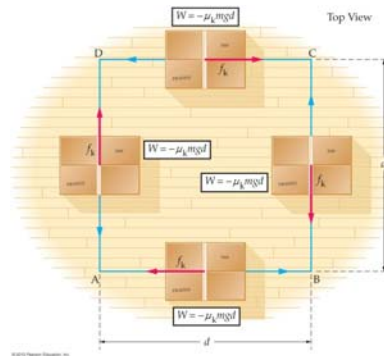
Note: To consider the potential energy of the spring, we CANNOT arbitrarily define where x=0 is.

Non-Conservative Force

Non-Conservative Force: Work depends on path.

TABLE 8-1 Conservative and Nonconservative Forces

Force	Section
Conservative forces	
Gravity	5-6
Spring force	6-2
Nonconservative forces	
Friction	6-1
Tension in a rope, cable, etc.	6-2
Forces exerted by a motor	7-4
Forces exerted by muscles	5-3



Mechanical Energy

Work done by conservative forces can be expressed as the difference in potential energy between the starting point and the end point. Work done by non-conservative forces cannot be so expressed. Differentiating these two types of forces, we can write

$$\frac{m}{2}v_f^2 - \frac{m}{2}v_o^2 = W = W_C + W_{NC} = (U_i - U_f) + W_{NC}$$

$$W_{NC} = (K_f - K_i) + (U_f - U_i) = \Delta K + \Delta U$$

Calling the sum of the kinetic energy and the potential energy the “total mechanical energy” E , we see that the work done by non-conservative forces on an object equals the increase in the total mechanical energy of that object:

$$W_{NC} = E_f - E_i.$$

In the absence of net non-conservative forces, the total mechanical energy of an object is “conserved”.

The Principle of Conservation of Mechanical Energy

The total mechanical energy ($E = K + U$) of an object remains constant as the object moves, provided that the net work done by external nonconservative forces is zero, $W_{NC} = 0J$.

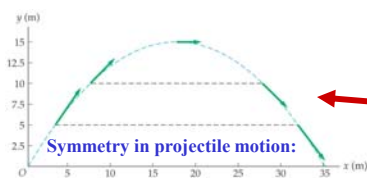
The most common form of potential energy is the gravitational potential energy. Therefore, the quantity

$$E = mgy + mv^2/2$$

is conserved in the absence of non-conservative forces.

Energy can neither be created nor destroyed, but can only be converted from one form to another.

Examples of Conserv. of Mech. Energy



In projectile motion, we observed incidentally that “same speed at same height”? Now we know why.

Prove that the landing speed of a projectile is independent of launch angle for a given height of launch.

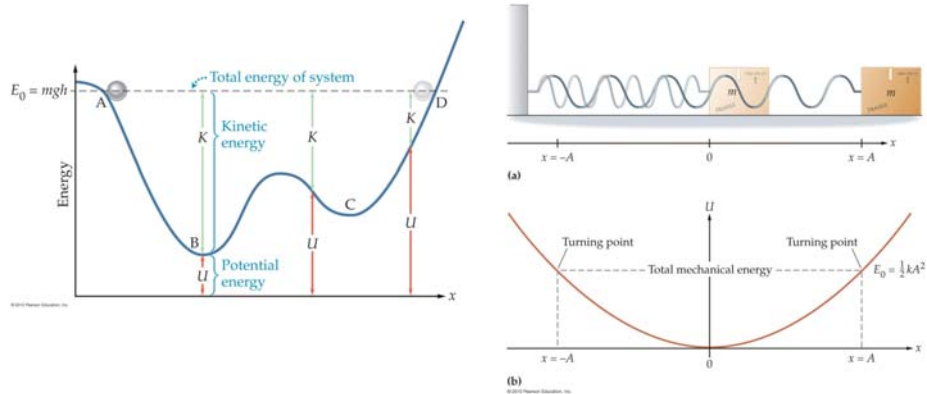
Mass-Spring

Projectile Motion

We proved this before. Now it's clear that it is just a consequence of energy conservation.

Potential Energy Curves and Equipotentials

Total mechanical energy $E = U + K = E_0$

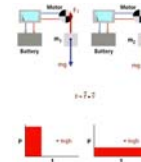


Power

Definition of Average Power

Average power \bar{P} is the average rate at which work W is done, and it is obtained by dividing W by the time t required to perform the work

$$\bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{\Delta t}$$



If the force is constant and points in the same direction as the displacement, the average power can be written as

$$\bar{P} = \frac{W}{\Delta t} = \frac{F \Delta x}{\Delta t} = F \bar{v}$$

SI Unit of Power: joule/s = watt (W)

Examples

37. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

61. A 75.0-kg cross-country skier is climbing a 3.0° slope at a constant speed of 2.00 m/s and encounters air resistance of 25.0 N. Find his power output for work done against the gravitational force and air resistance. (b) What average force does he exert backward on the snow to accomplish this? (c) If he continues to exert this force and to experience the same air resistance when he reaches a level area, how long will it take him to reach a velocity of 10.0 m/s?

Summary of Chapter 7

- Conservative forces conserve mechanical energy (gravity, spring)
- Nonconservative forces convert mechanical energy into other forms
- Conservative force does zero work on any closed path. Work done by a conservative force is independent of path
- Energy in the form of potential energy can be converted to kinetic or other forms
- Work done by a conservative force is the negative of the change in the potential energy **Gravity: $U = mgy$** **Spring: $U = \frac{1}{2} kx^2$**
- Mechanical energy is the sum of the kinetic and potential energies; it is conserved only in systems with purely conservative forces
- Non-conservative forces change a system's mechanical energy
- Work done by non-conservative forces equals change in a system's mechanical energy

Summary of Chapter 7

- The work is force times distance

$$W = (F \cos \theta)d = Fd \cos \theta$$

- Total work is equal to the change in kinetic energy:

$$W_{\text{total}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- Work done by a spring force: $W = \frac{1}{2}kx^2$

- Power is the rate at which work is done: $P = \frac{W}{t}$