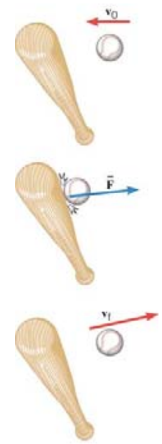
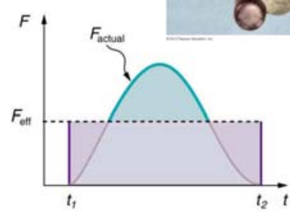
Chapter 8: Linear Momentum And Collisions

The impulse of a force is the product of the average force $\overline{F}$ and the time interval Δt during which the force acts:

$$\vec{I} = \overline{F}\ \Delta t$$





Impulse is a vector quantity and has the same direction as the average force.

I =F Δt = (m a) Δt = m (a Δt) = m Δv = Δ(mv)

Linear Momentum

The linear momentum p of an object is the product of the object's mass m and velocity v

p = m v

Linear momentum is a vector quantity and has the same direction as the velocity.

SI Unit of Momentum: kg m / s or N s

IMPULSE – MOMENTUM THEOREM

When a net force acts on an object, the impulse of the net force is equal to the change in momentum of the object.

Impluse = Change in momentum.

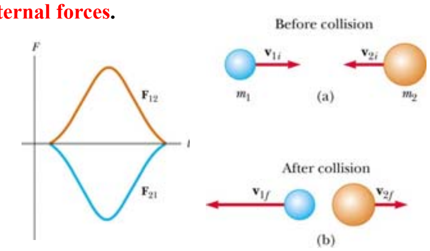
$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_o$$

Conservation of Linear Momentum

Often, it is convenient to define a "system" as a collection of particles or objects for the purpose of easier analysis of the motion (of the particles or of the system as a whole). When a system is defined, the forces acting on objects (within the system) can be distinguished into internal forces and external forces.

Internal forces: forces that the objects within the system exert on each other.

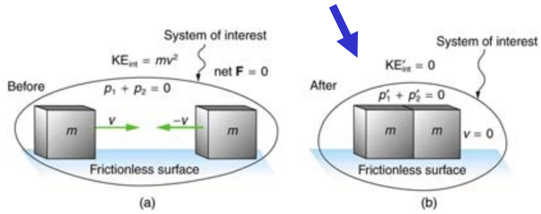
External forces: forces exerted on the objects by agents that are external to the system.



The (vector) sum of the linear momenta of all the objects in a system is the total linear momentum of the system. Because of Newton's action-reaction law, the total linear momentum of a system cannot be changed by internal forces. The change in the total linear momentum of a system equals the total impulse (a vector sum of all the impulses) due to EXTERNAL FORCES. If the sum of the total external forces is zero, the system is called an isolated system. For an isolated system, the total linear momentum is conserved, i.e. it remains constant.

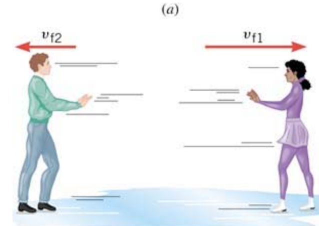
Conserv. Of Momentum (not nec. KE)

total KE decreases



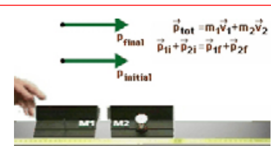
(a)

total KE increases



(b)

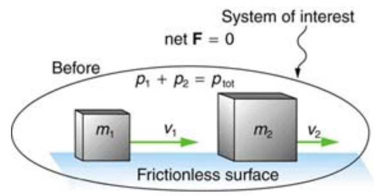
Momentum is conserved in all cases.



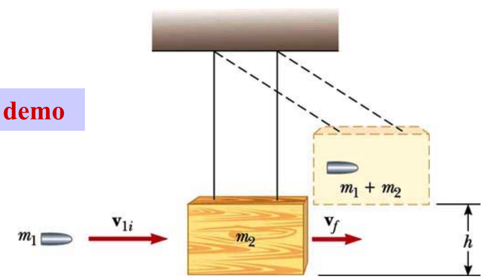
Elastic and Inelastic Collisions

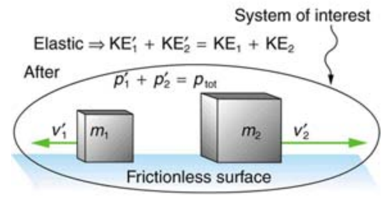
Elastic collision -- One in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision

Inelastic collision -- One in which the total kinetic energy of the system is not the same before and after the collision.



demo





Perfect 1D Inelas. Coll.

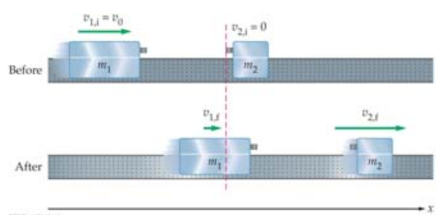
Collision in One Dimension (Elastic)

For any collision, momentum is conserved. We have

$$m_1 v_{f1} + m_2 v_{f2} = m_1 v_{o1} + 0$$

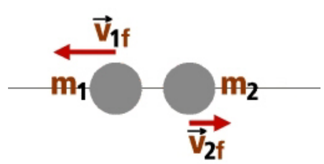
For elastic coll., KE is conserved.

$$\frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 = \frac{1}{2} m_1 v_{o1}^2 + 0$$



Solving for v_{f1} and v_{f2}, we get

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{o1} \qquad v_{f2} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{o1}$$

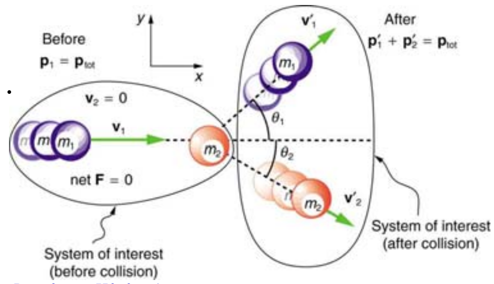


Collisions In Two Dimensions

Momentum is always conserved.

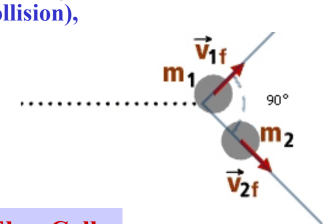
$$m_1 v_{f1x} + m_2 v_{f2x} = m_1 v_{o1x} + m_2 v_{o2x}$$

$$m_1 v_{f1y} + m_2 v_{f2y} = m_1 v_{o1y} + m_2 v_{o2y}$$



If kinetic energy is also conserved (i.e. elastic collision),

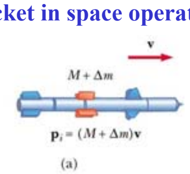
$$\frac{1}{2}m_1 v_{f1}^2 + \frac{1}{2}m_2 v_{f2}^2 = \frac{1}{2}m_1 v_{o1}^2 + \frac{1}{2}m_2 v_{o2}^2$$

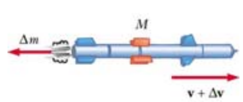


2D Elas. Coll.

System with Changing Mass: Rocket Propulsion

Cars, boats, airplanes accelerate by pushing against something (external).
Rocket in space operates by discharging part of itself at high speed.





$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$$

$$\Delta v = \frac{\Delta m}{M} v_e$$

$$\Delta p = v_e \Delta m$$

Thrust: $$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) v_e$$

$$\frac{v}{v_e} = \ln\frac{m_0}{m_r}$$

Example Problems

13. A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. Calculate the average force on the person (a) if he is stopped by a padded dashboard that compresses an average of 1.00 cm (b) if he is stopped by an air bag that compresses an average of 15.0 cm.

34. A battleship that is 6.00×10^7 kg and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) if the shell is fired straight aft, there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in internal kinetic energy (that is, for the ship and the shell).

45. Two identical pucks collide on an air hockey table. One puck was originally at rest. (a) If the incoming puck has a speed of 6.00 m/s and scatters to an angle of 30.0°, what is the velocity (magnitude and direction) of the second puck? (Use result $\theta_1 - \theta_2 = 90°$) (b) Confirm that the collision is elastic.

Summary of Chapter 8

- Linear momentum (a vector): $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
- Impulse = the change in momentum $\vec{I} = \vec{F}_{av}\,\Delta t = \Delta\vec{p}$
- Momentum is conserved if the net external force is zero
- Internal forces within a system always sum to zero
- Inelastic and Elastic collisions: kinetic energy conserved?
- One-Dimensional Elastic Collision

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{o1} \qquad v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{o1}$$

- Rocket Propulsion

$$\text{thrust} = \left(\frac{\Delta m}{\Delta t}\right) v$$