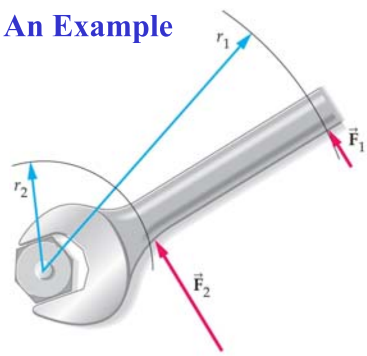
Chapter 9: Statics and Torque

An Example



Suppose we have an object pivoted at fixed frictionless axis. A force F_1, applied to the rod at a point r_1 for an angular displacement of $\Delta\theta$, after which the object has an angular velocity of ω. Since the radial component of force does no work (because there is no radial displacement), the kinetic energy is due to the work done by the tangential component of F_1. The displacement associated with the force F_1 is $r_1\Delta\theta$. If, instead of using F_1, we want to use a force F_2, at a distance r_2 from the rotational axis, to achieve the same acceleration, how much should F_2 be?

$$\text{K.E.} = F_1 r_1 \sin\theta_1 \,\Delta\theta = F_2 r_2 \sin\theta_2 \,\Delta\theta$$

$$F_1 r_1 \sin\theta_1 = F_2 r_2 \sin\theta_2 = \tau$$

Definition of Torque

Torque = (Magnitude of Force) * (Lever arm)

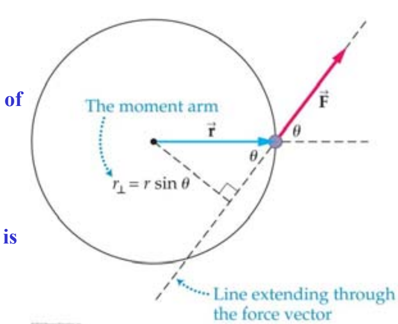
$$\tau = F\,d$$

Lever arm is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force.

$$\tau = r\,F \sin\theta$$

Obviously, the magnitude of a torque depends on where we assume the axis of rotation to be. F sinθ is the tangential component of the force. Note that a centripetal force leads to no torque.

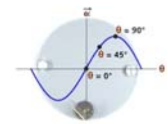
Unit of torque: newton-meter



Sign of torque

$\tau > 0$, if torque causes counterclockwise angular acceleration

$\tau < 0$, if clockwise angular acceleration



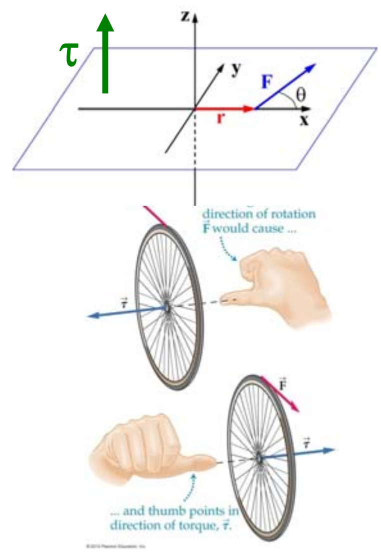
Vector Nature of Rotational Motion

Direction of torque is conventionally defined by the "right hand rule".

$$\vec{\tau} = \vec{r} \otimes \vec{F}$$

$\vec{r}$ points from the rotational axis to the location of the force.

The same convention is used to define angular velocity, angular acceleration, angular momentum, etc.

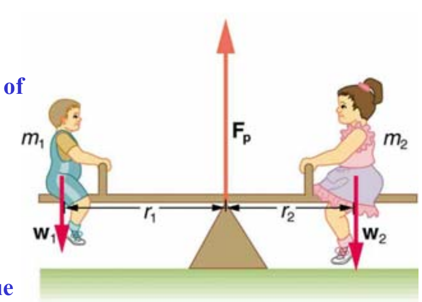


Conditions for Static Equilibrium

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero:

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{\tau} = 0$$

Note that if the net force is zero, the net torque becomes independent of the choice of rotational axis. In this case, we can use any rotational axis to sum up the net torque.

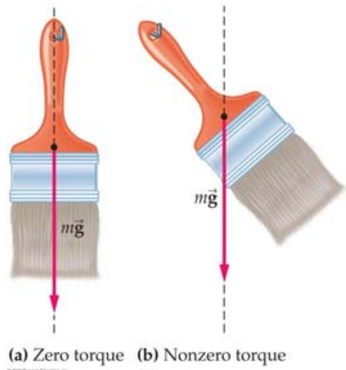


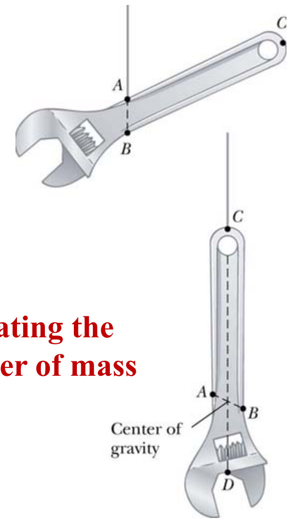
If the plank is weightless, what are r_1 and r_2?

What if the plank has a mass of M which is not uniformly distributed?

Center of Mass

For the purpose of calculating the torque due to the gravitational force on a rigid object, the total weight of the object can be regarded as concentrated on the "center of gravity" of this rigid body.

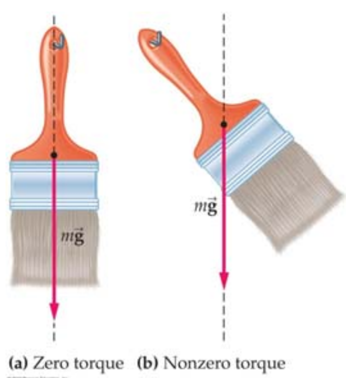




Locating the center of mass

Stable and Meta-stable Equilibrium

A brush hanging on a hook finds an equilibrium when its center of gravity is directly below the position of the hook.

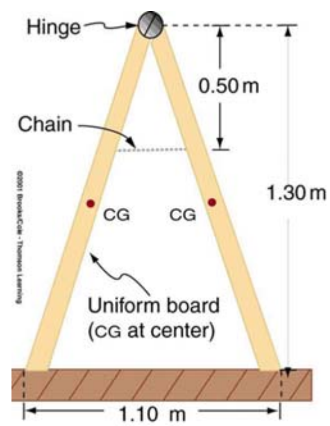


It is still in equilibrium, if the brush were carefully balanced brush-side up and resting on the tip of its handle. Why is it now unstable?

Examples

9. A person carries a plank of wood 2.00 m long with one hand pushing down on it at one end with a force F_1 and the other holding it up at .500 m from the end of the plank with force F_2. If the plank has a mass of 20.0 kg and its center of gravity is at the middle of the plank, what are the magnitudes of the forces F_1 and F_2?

14. A sandwich board advertising sign is constructed as shown. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewald. (b) What force is exerted by each side on the hinge?



Summary of Chapter 9

•**Torque :** $\tau = rF\sin\theta$

• **Newton's second law for rotation:**

• **Static equilibrium: the total force and the total torque acting on the object must be zero.**

• **An object balances when it is supported at its center of mass.**

• **Rotational quantities are vectors that point along the axis of rotation, with the direction given by the right-hand rule.**