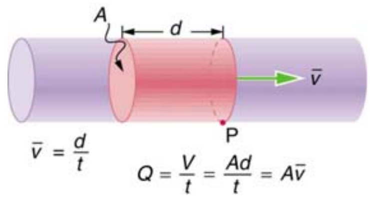
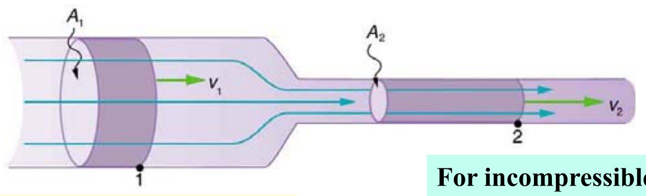
Chapter 12 Fluid Dynamics

Equation of Continuity



The mass flow rate (ρAv) has the same value at every position along a continuous tube. For two positions along such a tube



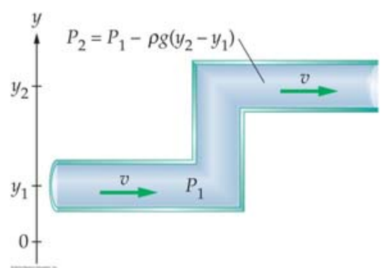
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

For incompressible fluids,

$$A_1 v_1 = A_2 v_2$$

Bernoulli's Equation

In the steady flow of a nonviscous, incompressible fluid of density ρ, the pressure P, the fluid speed v, and the elevation y at any two points (1 and 2) are related. Consider a streamline connecting the two locations, the work done per unit time is



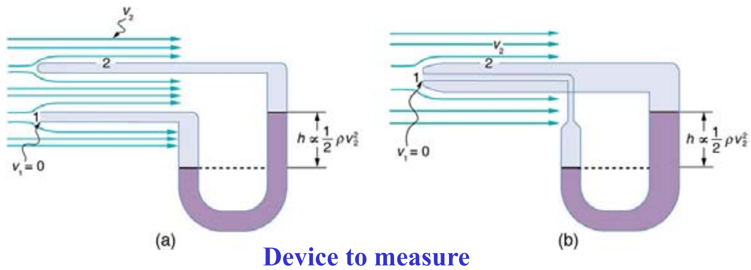
$$dW = P_1 dV - P_2 dV$$

which is equal to the change in total mechanical energy

$$dW = \tfrac{1}{2}(\rho dV)v_2^2 + (\rho dV)gy_2 - \tfrac{1}{2}(\rho dV)v_1^2 - (\rho dV)gy_1$$

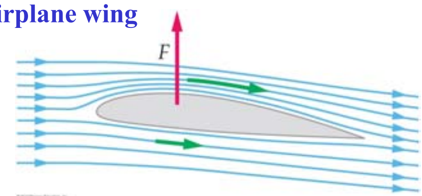
$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 = const.$$

Bernoulli’s Equation Examples

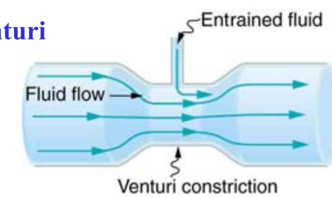


Device to measure fluid velocity

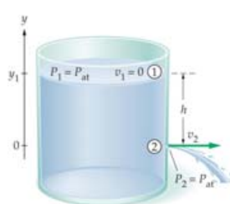
Cross section of an airplane wing

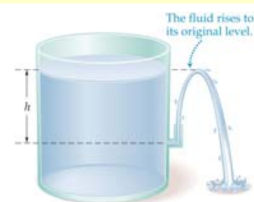


Venturi



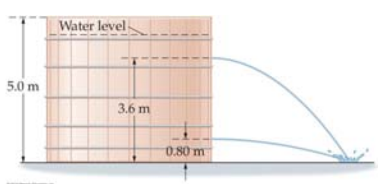
Toricelli’s Law





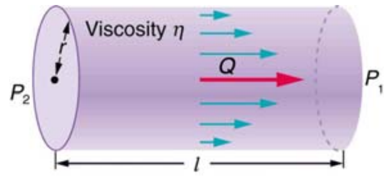
$$P_{at} + \frac{1}{2}\rho \cdot 0^2 + \rho g \cdot h = P_{at} + \frac{1}{2}\rho v_2^2 + \rho g \cdot 0$$

$$v_2 = \sqrt{2gy_1}$$



The water tank is open to the atmosphere and has two holes in it, one 0.80 m and one 3.6 m above the floor on which the tank rests. If the two streams of water strike the floor in the same place, what is the depth of water in the tank?

Viscous Flow



In viscous fluid, there is "friction" between regions with different velocity. As a result, a pressure difference needs to be present to maintain a steady flow of fluid. For a cylindrical pipe, the coefficient of viscosity η is the ratio between the pressure difference and vL/A, where v is the average fluid velocity

$$P_1 - P_2 = 8\pi\eta\frac{vL}{A}$$

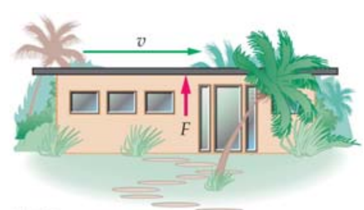
conventional unit: 1 poise = 0.1 N s/m^2

Volume flow rate
$$\frac{dV}{dt} = vA = \frac{(P_1 - P_2)A^2}{8\pi\eta L} = \frac{(P_1 - P_2)\pi r^4}{8\eta L}$$

Poiseuille's Equation

Example Problems

21. Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s. Approximately what is the force due to the Bernoulli effect on a roof having an area of 220m^2?



28. A sump pump is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of 3.00x10^5 N/m^2. (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

Chapter 12 Summary

- **Equation of continuity:** $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$
- **Bernoulli's equation:**

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

- **A pressure difference is required to keep a viscous fluid moving:**

$$\frac{\Delta V}{\Delta t} = \frac{(P_1 - P_2)\pi r^4}{8\eta L}$$