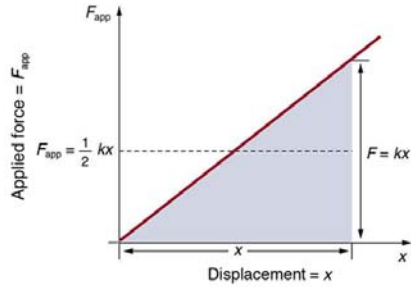# Chapter 16: Oscillatory Motion and Waves

Method A

$$W = \frac{1}{2} bh = \frac{1}{2} kxx$$

$$W = \frac{1}{2} kx^2$$

Method B

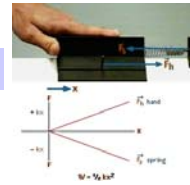$$W = f \cdot x = \left(\frac{1}{2} kx\right)(x)$$

$$W = \frac{1}{2} kx^2$$

Hooke's Law (revisited)

Spring-block

$$F = - k\,x$$

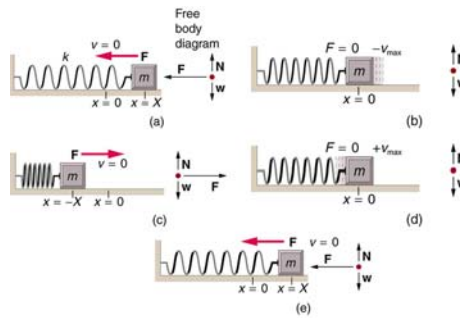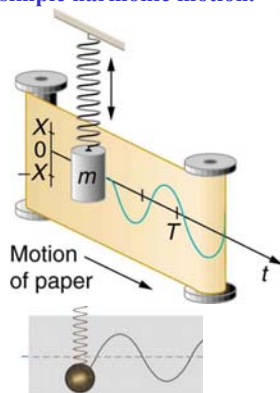**Tthe elastic potential energy of a stretched or compressed spring is**

$$PE_{elastic} = kx^2/2$$

Note: To consider the potential energy of the spring, we CANNOT arbitrarily define where x=0 is.
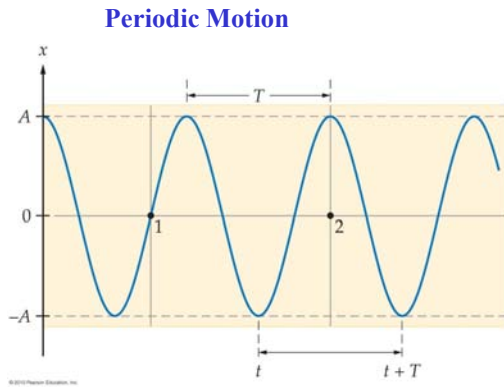
Simple Harmonic Motion (SHM)

When the restoring force has the mathematical form given by F = -kx (Hooke's Law), the resultant periodic motion is referred to as "simple harmonic motion."

Position versus time in SHM can be represented by a sinusoidal function
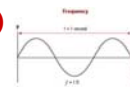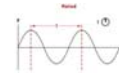
$$x = A\cos\left(\frac{2\pi t}{T}\right)$$
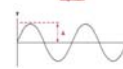
1

Period, Frequency, Wave Length, Speed

Periodic Motion

Period (T): time required for one complete cycle of periodic motion
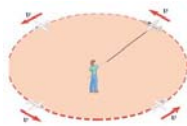
Frequency $f = \dfrac{1}{T}$

unit: 1/s (=Hz)

amplitude (A)

We already knew an example of SHM:

Uniform Circular Motion!

$$F_{cp} = mv^2 / R = m\omega^2 R$$

$$F_x = -m\omega^2 R \cos\theta = -(m\omega^2)\, x$$

$$k_{eff} = m\omega^2$$
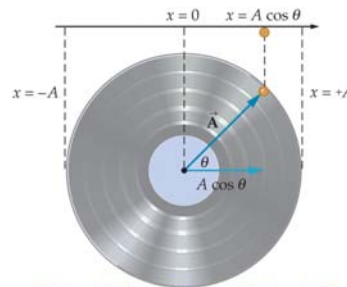
If we analyze the x-direction motion of an object in uniform circular motion, we see that the x-component of centripetal force on the object is proportional to its displacement in the x-direction (and with a negative sign). In other words, the x-direction force experienced by an object in uniform circular motion (ω) is identical to an object bound to the end of a spring, with a spring constant of $m\omega^2$, in a simple harmonic oscillation.

Kinematics of Simple Harmonic Motion

Since the x-component of the force is identical, both motions (in the x-direction) must be identical. To describe the motion of an object on a spring, we only need to analyze the 1-D projection of a object in uniform circular motion.

$$x = A\cos\theta = A\cos(\omega t + \theta_0)$$

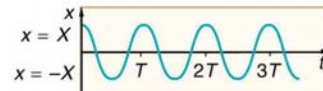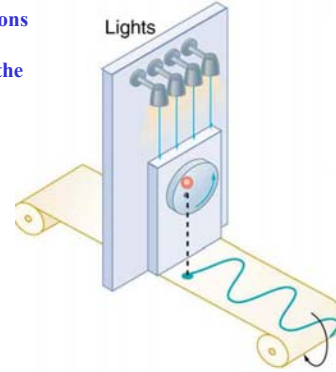$$v_x = -A\omega\sin\theta = -A\omega\sin(\omega t + \theta_0)$$

$$a_x = -A\omega^2\cos\theta = -A\omega^2\cos(\omega t + \theta_0)$$

Plugging these expressions into our original equation of F = -k x = m a , we identify that

$$\omega = \sqrt{\frac{k}{m}}$$

Note: Frequency of SHM is independent of amplitude.

"Recoil Time" of a spring

SHM compared with uniform circular motion

More about Simple Harmonic Motion

Conservative Force: Total mechanical energy is conserved:

$$E_{Total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$
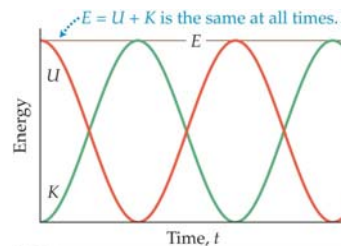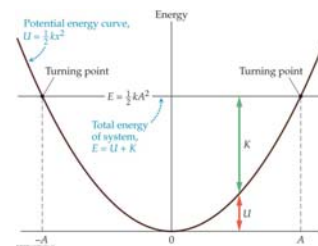
$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$v_{max} = A\omega$$

$$a_{max} = kA/m = A\omega^2$$

$$f = \frac{\omega}{2\pi}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$


E = U + K is the same at all times.

The Simple Pendulum

The gravitational force acting on a mass m on a string with length of L, when it is displaced a small distance s from its equilibrium position, has an approximate horizontal force component of

$$F_h = - (mg/L) \, s \quad,$$

acting in a direction to push the mass back toward the equilibrium position. So the force acting on the pendulum is approximately the same as that from a spring with a spring constant of k = (mg/L).
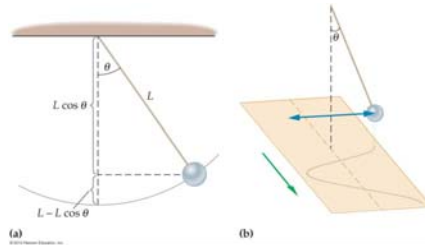
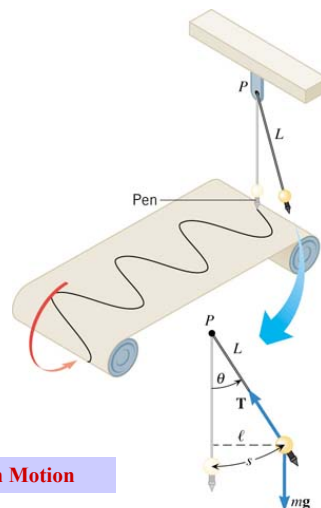

Pendulum Cf. Spring-Mass

Motion of The Pendulum

Since the restoring force on the pendulum is approximately proportional to the displacement of the pendulum, the resultant motion is approximately "simple harmonic". The angular frequency of the pendulum motion is

$$\omega = \sqrt{\frac{g}{L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

T is independent of m!

Pendulum Motion

SHM Problems

18. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

38. A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?
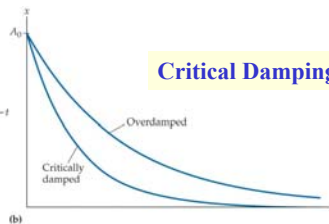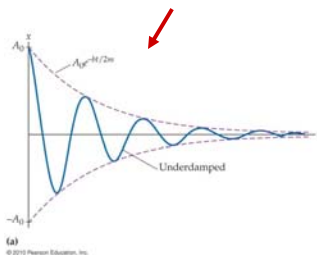
Damped Oscillations

Assume that for a simple harmonic motion, the damping (force) can be represented by $\vec{F} = -b\vec{v}$

It can be shown that the oscillation amplitude decays with time for "under-damped" systems
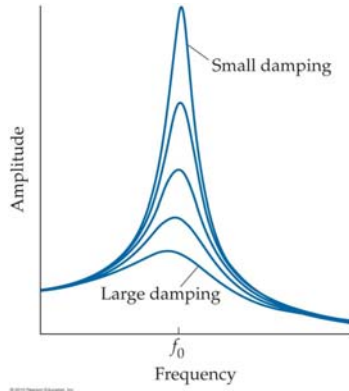
$$A(t) = A_0\, e^{-bt/2m}$$

$$x = A_0\, e^{-bt/2m} \cos(\omega t + \theta_0)$$

Damped Oscillations

Critical Damping: $\dfrac{b}{2m} = \omega = \sqrt{\dfrac{k}{m}}$

Driven Oscillations

Suppose a periodic driving force acts on a simple harmonic oscillator

$$F_{drive} = C \cos(\omega_{dr} t)$$

After some "transient period", a steady state will be reached

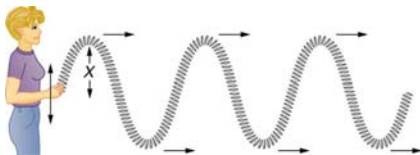$$x = A \cos(\omega_{dr} t + \theta_c)$$

Without damping, the amplitude will "diverge" at the natural angular frequency of the system $\omega_0 = \sqrt{k/m}$

With small damping, a resonance takes place near the natural frequency, at
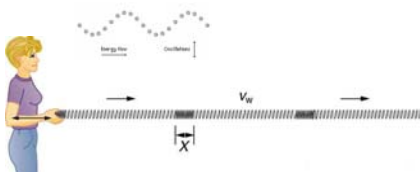
$$\omega_{res}^2 = \omega_0^2 - b^2 / 2m^2$$

Waves

Waves: A wave is a traveling disturbance. It carries energy but involves no net transportation of material.
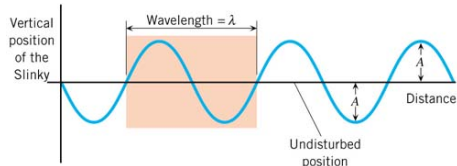

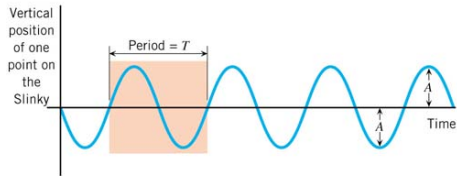A **transverse wave** has disturbance perpendicular to the direction of the wave.

A **longitudinal wave** has disturbance parallel to the direction of the wave.
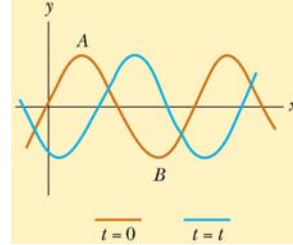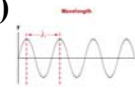
Periodic Waves

Vertical position of the Slinky

Wavelength = λ

Distance

A

A

Undisturbed position

(a) At a particular time

Vertical position of one point on the Slinky

Period = T

Time

A

A

(b) At a particular location

wavelength (λ)

speed (v = λ/T = λf)

A sinusoidal wave is one whose disturbance as a function of time (or position) can be described by a sine or cosine function.

Harmonic (Sinusoidal) Waves

$t = 0$ $x = 0$

$t = T/4$ $x = \lambda/4$

$t = T/2$ $x = \lambda/2$

$t = 3T/4$ $x = 3\lambda/4$

$t = T$ $x = \lambda$
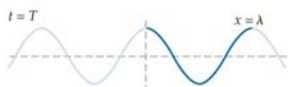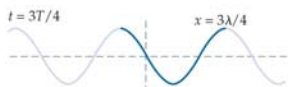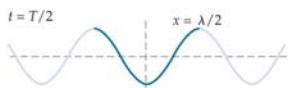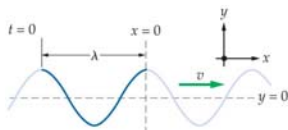
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A sinusoidal wave is one whose disturbance as a function of time (or position) can be described by a sine or cosine function.

$$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}\left\{x - x_0 - \lambda\frac{t}{T}\right\}\right)$$

$$y(x,t) = A\cos\left(kx - \omega t\right)$$

7

Speed of Wave on a String

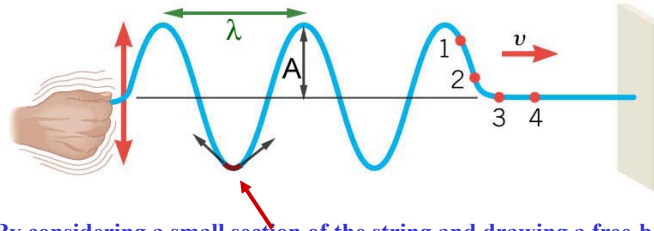What is the speed of a transverse wave traveling on a string with a linear density of mass of m/L and a tension of F ? Assume the amplitude to be small, namely, A << λ. What law(s) do we use to determine the speed of wave on a string?
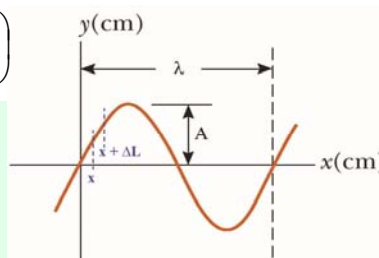

By considering a small section of the string and drawing a free-body diagram, we find that the net force on the small section is not zero, because the tensions on the two ends of the small section are pointed at slightly different directions. Therefore, we can determine the speed of wave by using Newton's Second Law!

Speed of Wave on a String

assume sinusoidal wave, at a particular time

$$y(x) = A \sin\left(\frac{2\pi x}{\lambda}\right)$$

$$slope]_x = \frac{\Delta y}{\Delta x}$$

$$= \frac{A \sin[2\pi(x+\Delta x)/\lambda] - A \sin[2\pi x/\lambda]}{\Delta x}$$

$$= \frac{2\pi A \cos(2\pi x/\lambda)}{\lambda}$$

$$net\ force = F * slope]_{x+\Delta L} - F * slope]_x$$

$$= \frac{2\pi AF \cos[2\pi(x+\Delta L)/\lambda]}{\lambda} - \frac{2\pi AF \cos(2\pi x/\lambda)}{\lambda}$$

$$= -\frac{4\pi^2 F\Delta L}{\lambda^2} A \sin(2\pi x/\lambda)$$

$$k_{eff.} = 4\pi^2 F\Delta L\lambda^{-2}$$

$$\omega = \sqrt{\frac{4\pi^2 F\Delta L\lambda^{-2}}{(m/L)\Delta L}}$$

$$v = \frac{\omega}{2\pi}\lambda = \sqrt{\frac{F}{(m/L)}}$$

Interference of Waves

The Principle of Linear Superposition

When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.

Linear Superposition

(a) Overlap begins

(b) Total overlap; the slinky has twice the height of either pulse

(c) The receding pulses

(a) Overlap begins

(b) Total overlap

(c) The receding pulses
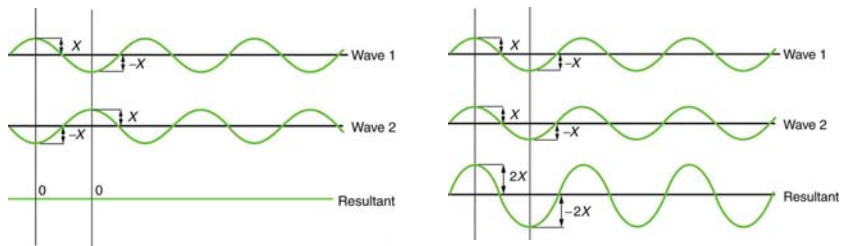
Reflection of Waves

- **When a traveling wave reaches a boundary, all or part of it is reflected**
- **When reflected from a free end, the pulse is not inverted**
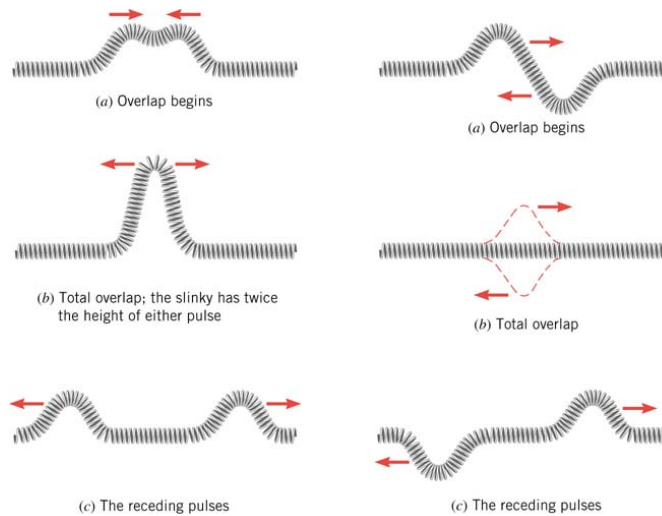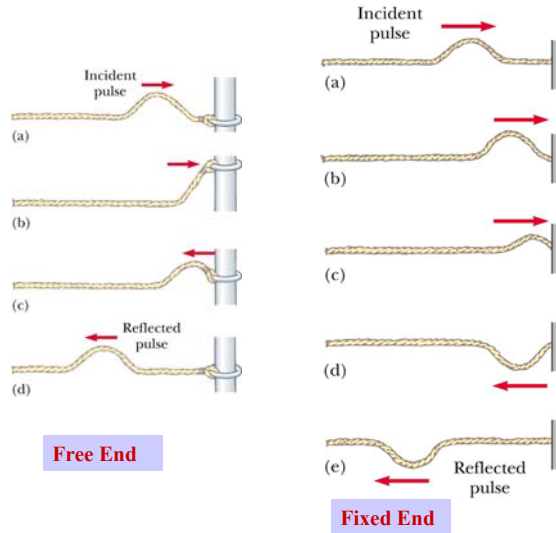- **When it is reflected from a fixed end, the wave is inverted**

Free End

Fixed End

Standing Waves

Standing wave is due to the interference between two identical waves traveling in opposite directions.   The fixed end point of a string acts as a reflection boundary, reversing the direction and the sign of displacement of the incoming wave. Nodes in a standing wave are **separated by $\lambda/2$. (not λ !)** Note that the amplitude of the lobes can exceed the amplitude of the driving source.

Node   **Antinode**
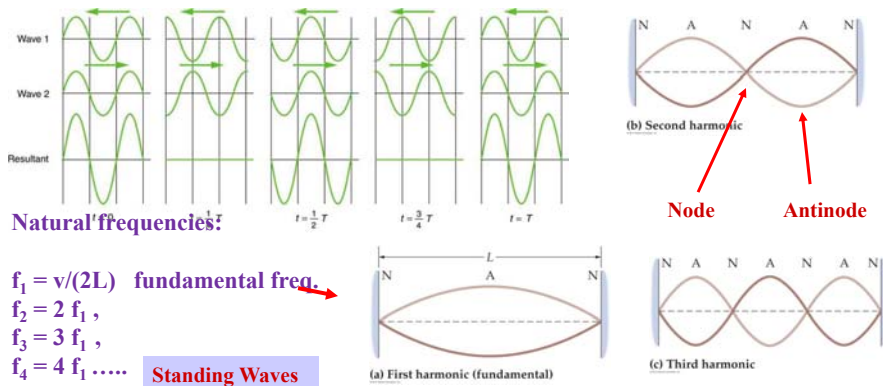
Natural frequencies:

$f_1 = v/(2L)$ fundamental freq.
$f_2 = 2 f_1$,
$f_3 = 3 f_1$,
$f_4 = 4 f_1$ .....

Standing Waves

Interference of Waves
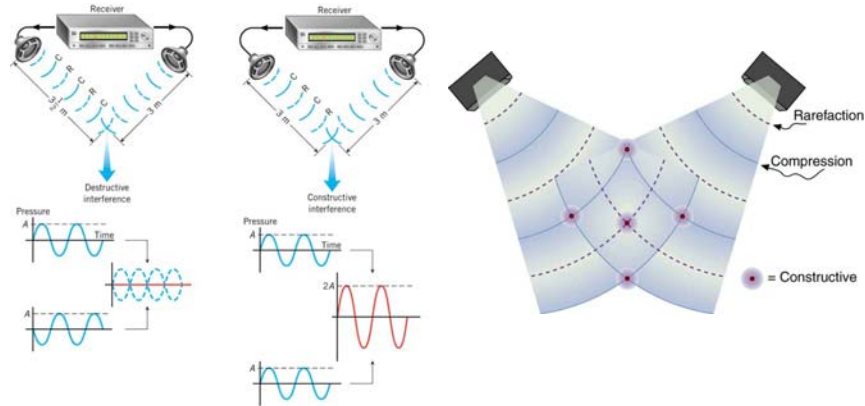
Only coherent sources can produce interference effect. For two wave sources vibrating in phase, a difference in path lengths that is zero or an integer number {1,2,3,..} of wavelengths leads to constructive interference; a difference in path lengths that is a half-integer number {0.5, 1.5, 2.5, ...} of wavelengths leads to destructive interference.
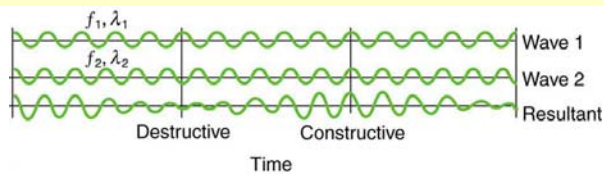
Beats

Two oscillations close in frequency leads to periods of approximate constructive interference and destructive interference. The volume of the sound appears to flip between "large" and "small", forming "beats".

What is the beat frequency?

Suppose at time t=0, the two primary sounds are "in phase" (maximum). We want to know the next time when the two sounds are again "in phase". The time elapsed, T_{beat}, is then obviously the "period" of the beats. This happens when the sound with the higher frequency (f_1) has gone one full cycle more than the slower wave (f_2).

$$f_1 T_{beat} - f_2 T_{beat} = 1$$
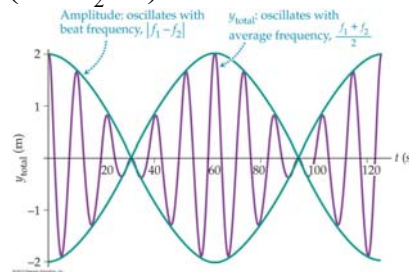$$T_{beat} = (f_1 - f_2)^{-1}$$
$$f_{beat} = f_1 - f_2$$

Beats

$$f_{beat} = | f_1 - f_2 |$$

11

$$y_1 = A\cos(2\pi f_1 t) \qquad y_2 = A\cos(2\pi f_2 t)$$

$$y_{total} = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$$

$$= A\cos[2\pi(\tfrac{f_1+f_2}{2} + \tfrac{f_1-f_2}{2})t] + A\cos[2\pi(\tfrac{f_1+f_2}{2} - \tfrac{f_1-f_2}{2})t]$$

$$= 2A\cos(2\pi \tfrac{f_1+f_2}{2} t) \cdot \cos(2\pi \tfrac{f_1-f_2}{2} t)$$

$$\boxed{f_{beat} = |f_1 - f_2|}$$

Power and Intensity of Waves

Intensity of a wave, I, is the power, P, per area that passes perpendicularly through a surface:

I = P / A

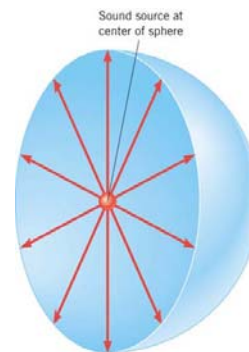**If a source emits waves uniformly in all directions, the intensity at distance r away from the source is**

I = P / (4πr²)

Therefore, the ratio of intensities at two spherical surfaces is

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

At very large distances from any source, or for waves traveling in confined channels, wave fronts are almost parallel. This is known as a plane wave.

12

58. The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

64. The low-frequency speaker of a stereo set has a surface area of 0.05 m² and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity 0.1 W/m²?

Chapter 16 Summary

• **Period of a mass on a spring:** $\quad T = 2\pi\sqrt{\dfrac{m}{k}}$

• **Total energy in simple harmonic motion:** $\quad E = \frac{1}{2}kA^2$

• **Period of a simple pendulum:** $\quad T = 2\pi\sqrt{\dfrac{L}{g}}$

• **Wave speed:** $\quad v = \lambda f$

• **Speed of a wave on a string:** $\quad v = \sqrt{\dfrac{F}{\mu}}$

• **Intensity of sound:** $\quad I = \dfrac{P}{A}$

• **Resonance occurs when the driving frequency is close to the natural frequency of the system**

Summary

• **When two or more waves occupy the same location, their displacements are added. Constructive and destructive.**

Standing waves on a string: $f_n = nf_1 = n(v/2L)$

$$\lambda_n = \lambda_1/n = 2L/n$$

• **Beat frequency:** $f_{\text{beat}} = |f_1 - f_2|$