Chap. 4 Equilibrium of Rigid Bodies

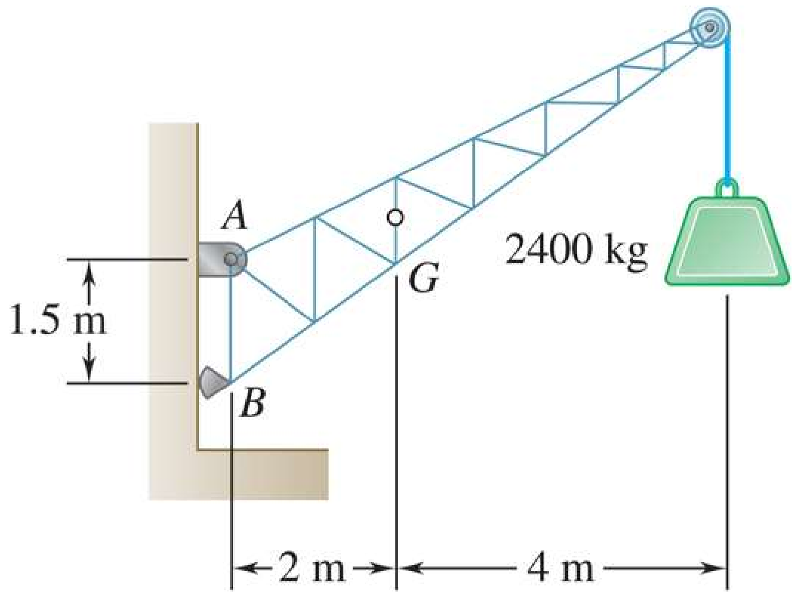
- For a rigid body, the condition of static equilibrium means that the *body under study* does not translate or rotate under the given loads that act on the body

- The necessary and sufficient conditions for the static equilibrium of a body are that the forces sum to zero, and the moment about any point sum to zero:

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum\left(\vec{r} \times \vec{F}\right) = 0$$

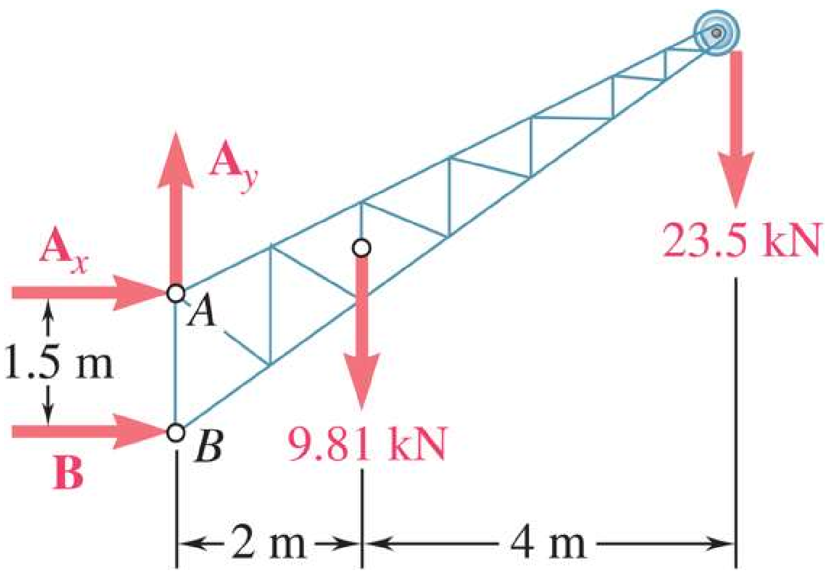
- Equilibrium analysis can be applied to two-dimensional or three-dimensional bodies, but the first step in any analysis is the creation of the *free body diagram*

Free-Body Diagram

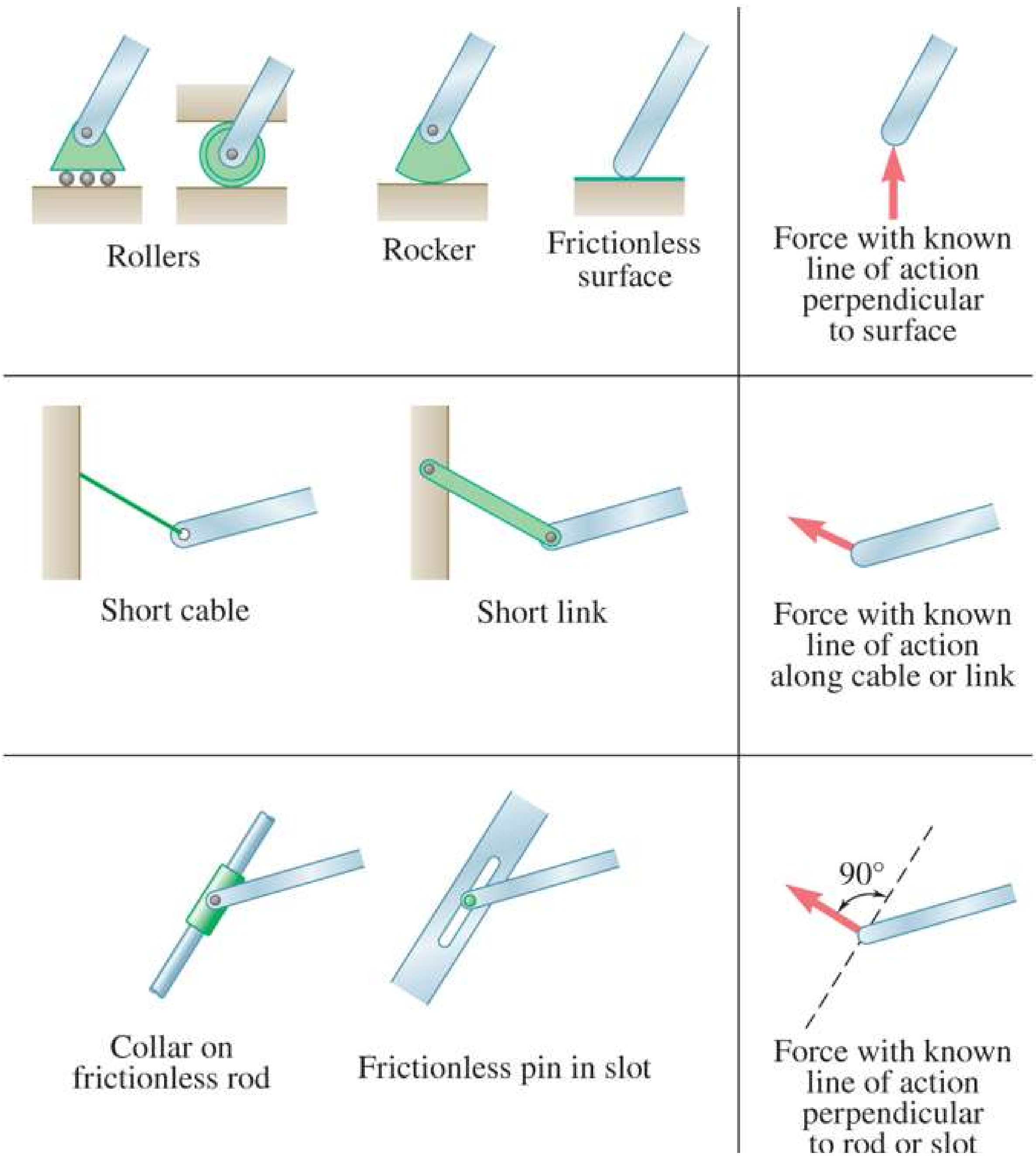


The first step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free body diagram*.

- Select the body to be analyzed and detach it from the ground and all other bodies and/or supports.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown forces from reactions of the ground and/or other bodies, such as the supports.
- Include the dimensions, which will be needed to compute the moments of the forces.



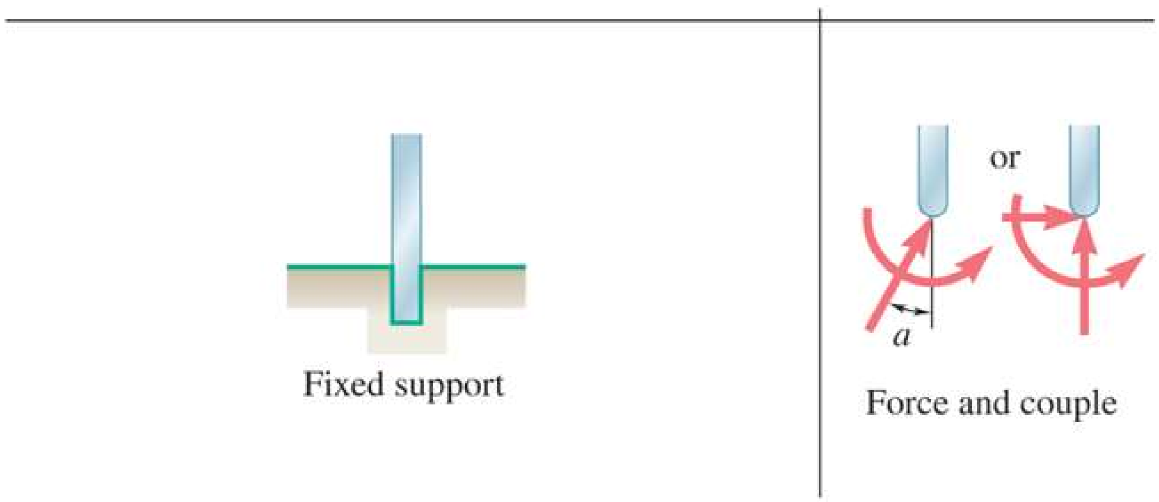
Reactions at Supports and Connections for a Two-Dimensional Structure



- Reactions equivalent to a force with known line of action.

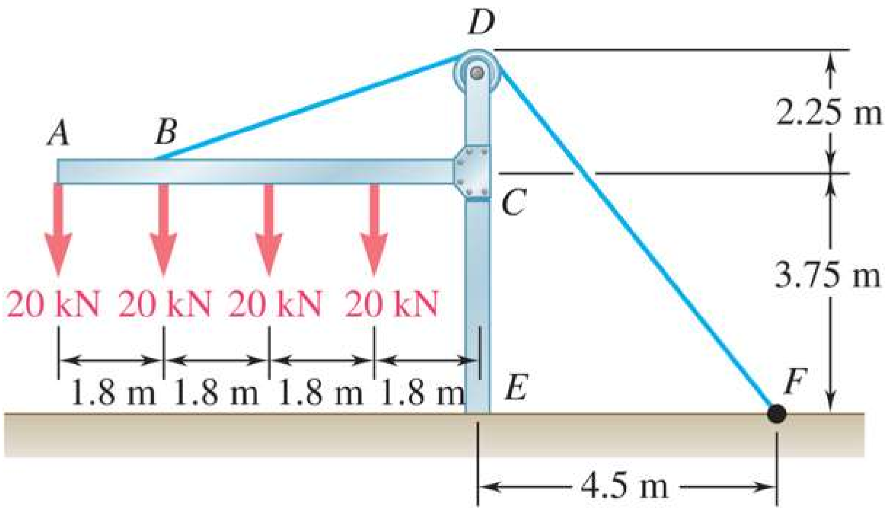
Reactions at Supports and Connections for a Two-Dimensional Structure

- Reactions equivalent to a force of unknown direction and magnitude.

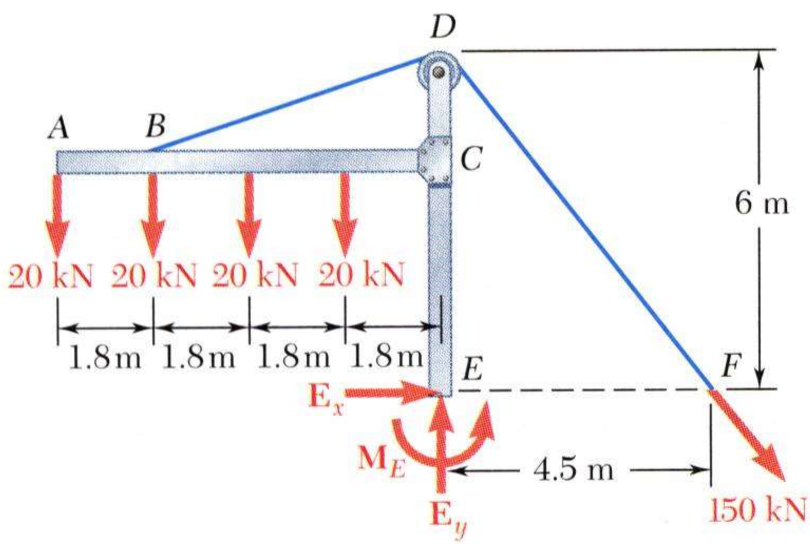


- Reactions equivalent to a force of unknown direction and magnitude and a couple.of unknown magnitude

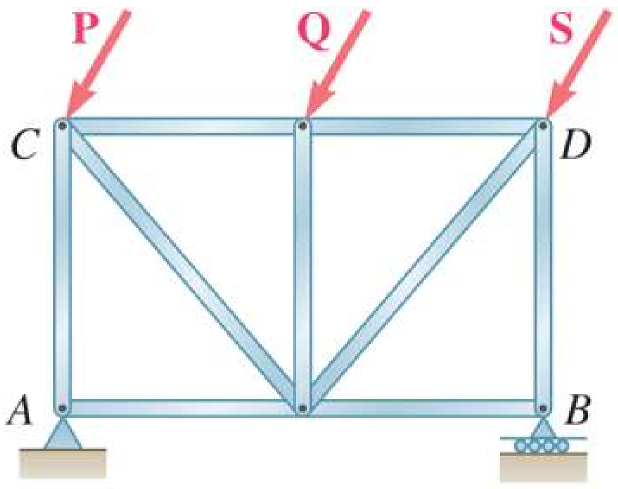
Practice



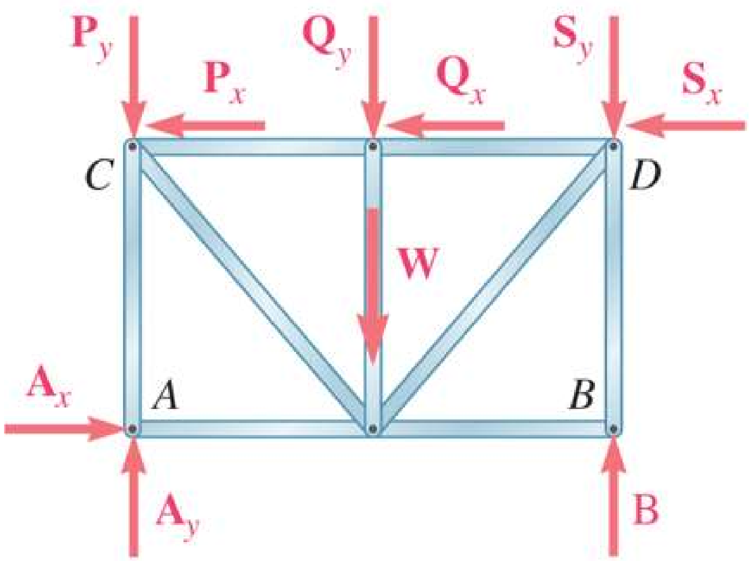
The frame shown supports part of the roof of a small building. Your goal is to draw the free body diagram (FBD) for the frame. (Neglect the weight of objects.)



Equilibrium of a Rigid Body in Two Dimensions



(a)



(b)

- For known forces and moments that act on a two-dimensional structure, the following are true:

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

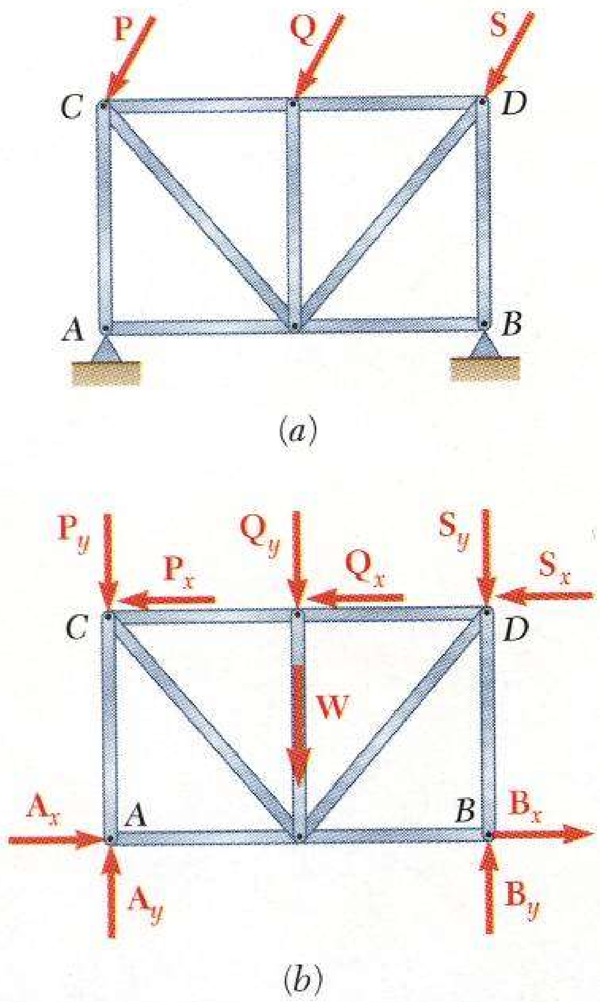
where A can be any point in the plane of the body.

- The 3 equations can be solved for no more than 3 unknowns.

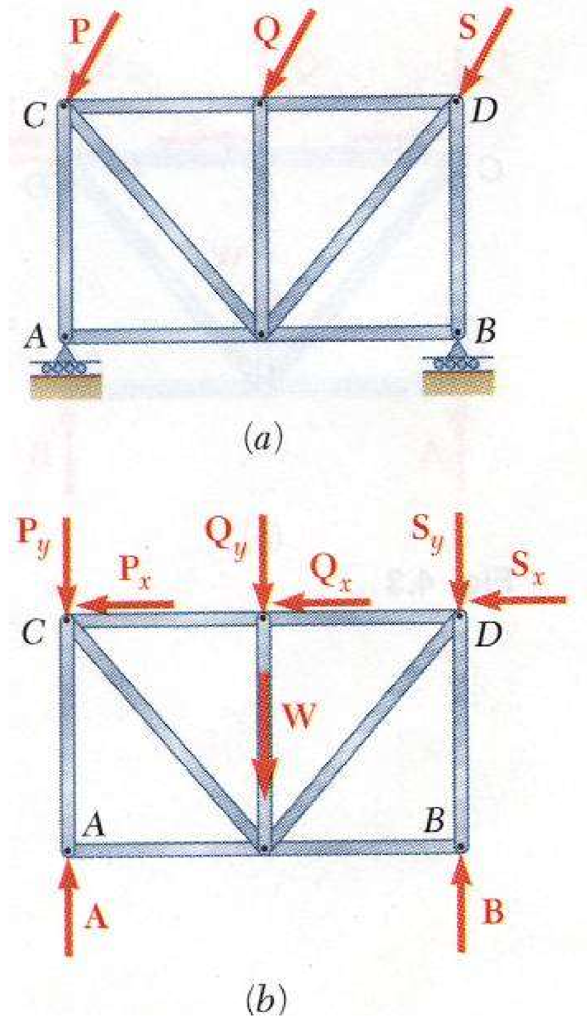
- The 3 equations cannot be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

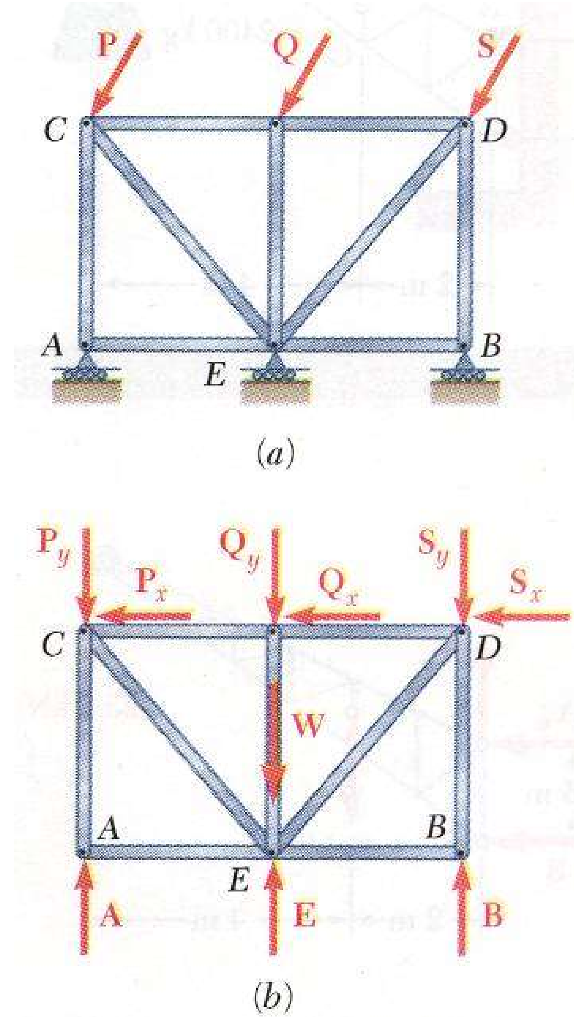
Statically Indeterminate Reactions



- More unknowns than equations

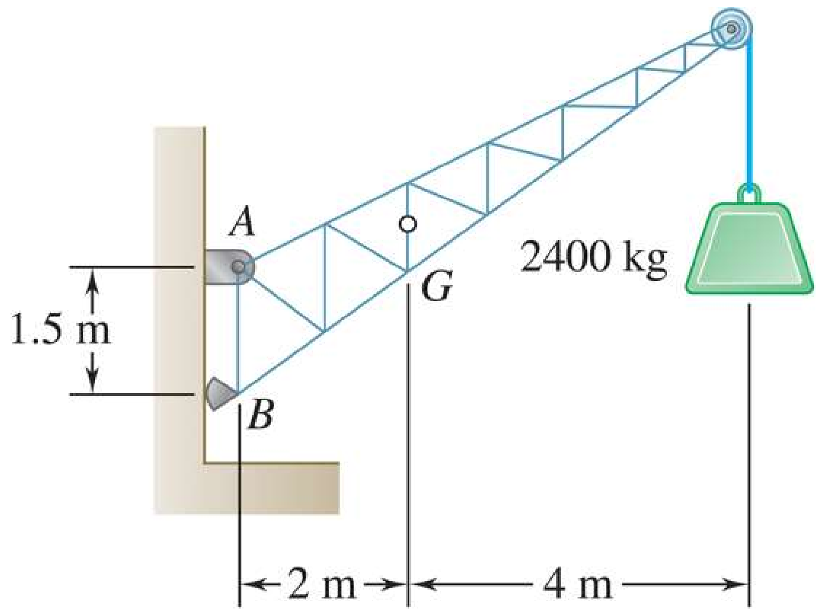


- Fewer unknowns than equations, partially constrained



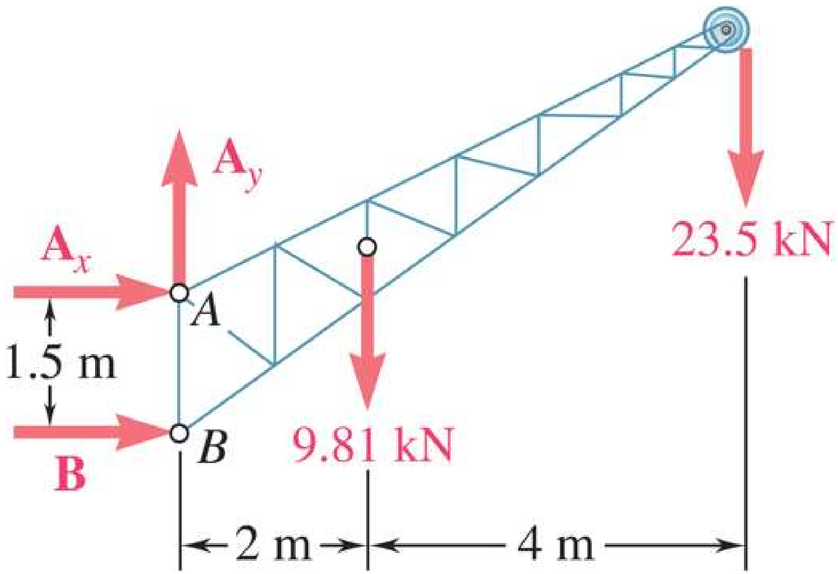
- Equal number unknowns and equations but improperly constrained

Sample Problem 4.1



A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G.

Determine the components of the reactions at A and B.



- Determine B

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$B = +107.1\,\text{kN}$$

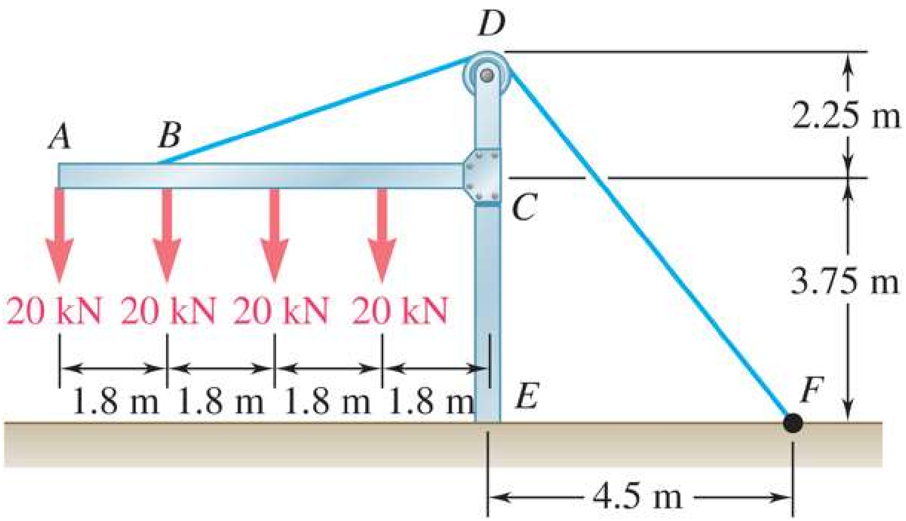
- Determine the reactions at A

$$\sum F_x = 0: \quad A_x + B = 0 \qquad A_x = -107.1\,\text{kN}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

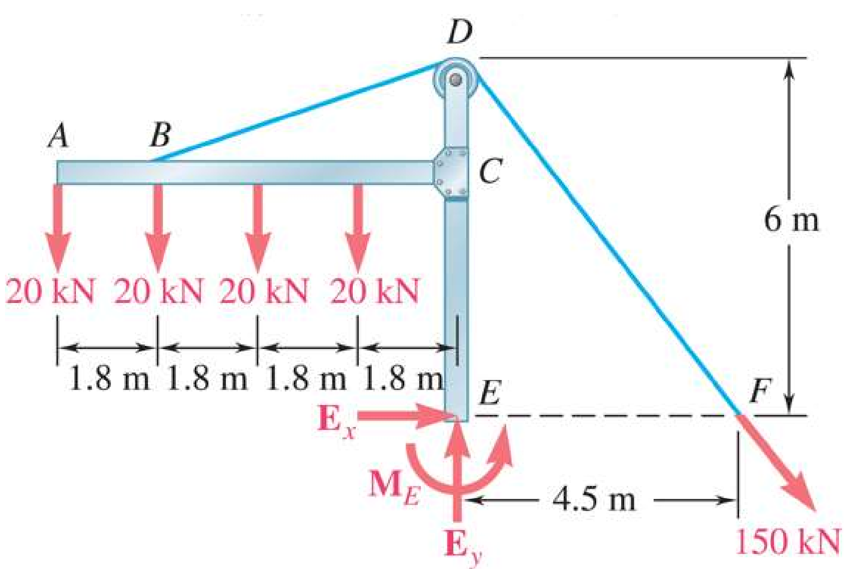
$$A_y = +33.3\,\text{kN}$$

Sample Problem 4.4



The frame supports part of the roof of a small building. The tension in the cable is 150 kN.

Determine the reaction at the fixed end *E*.

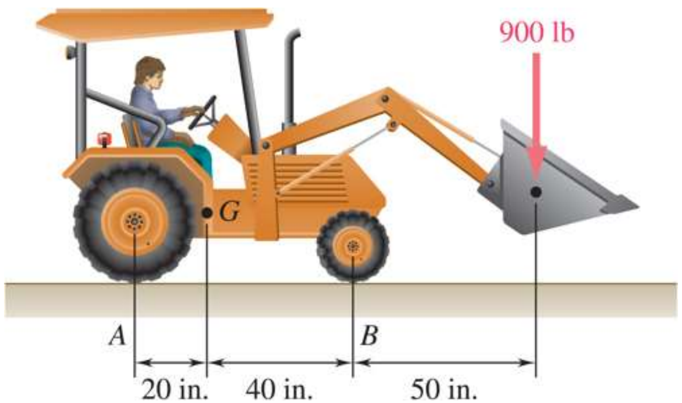


$$\sum F_x = 0: \quad E_x + \frac{4.5}{7.5}(150\,\text{kN}) = 0$$

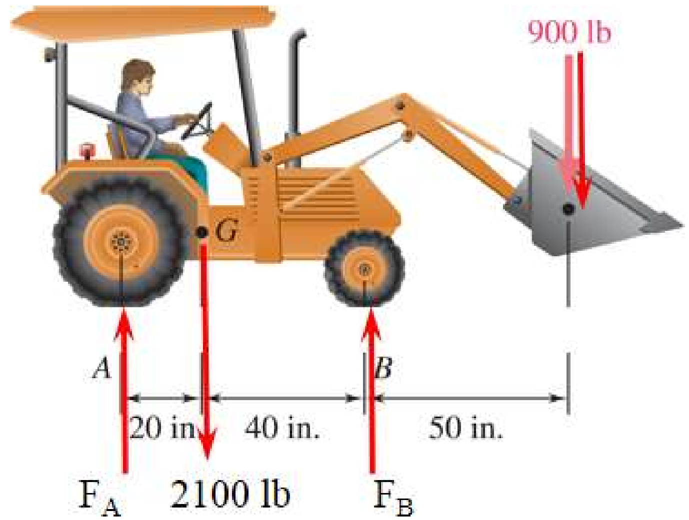
$$\sum F_y = 0: E_y - 4(20\,\text{kN}) - \cos 36.9^o(150\,\text{kN}) = 0$$

$$\sum M_E = 0: \quad +20\,\text{kN}(7.2\,\text{m}) + 20\,\text{kN}(5.4\,\text{m}) + 20\,\text{kN}(3.6\,\text{m}) + 20\,\text{kN}(1.8\,\text{m}) - \frac{6}{7.5}(150\,\text{kN})4.5\,\text{m} + M_E = 0$$

Practice



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two rear wheels and two front wheels



$$\sum M_B = 0.$$

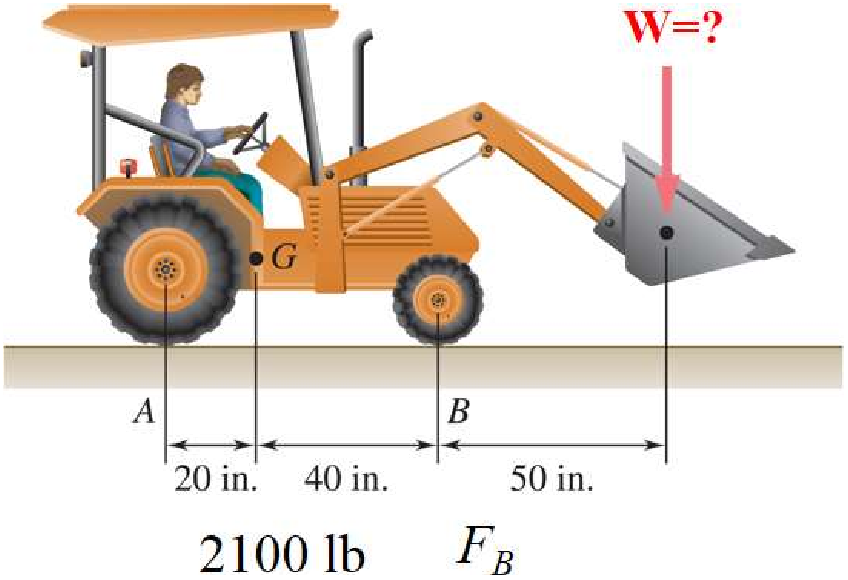
$-F_A$ (60 in.)+2100lb (40 in.)-900 lb (50 in.)=0

F_A=650 lb, so the reaction *at each wheel* is 325 lb

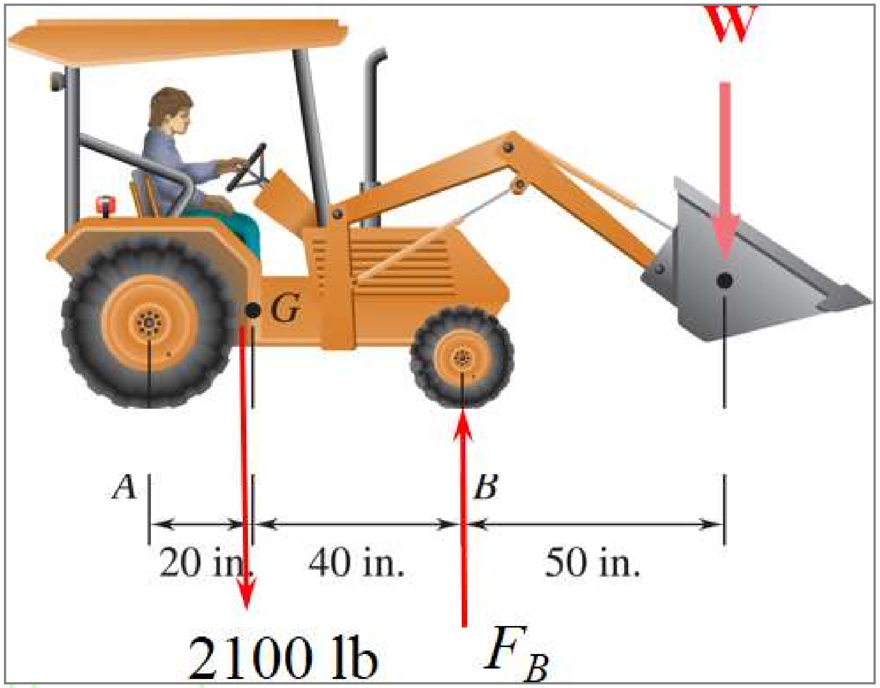
$\Sigma F_y = 0$ $\Rightarrow F_B = 2350$ lb,

or 1175 lb at each front wheel

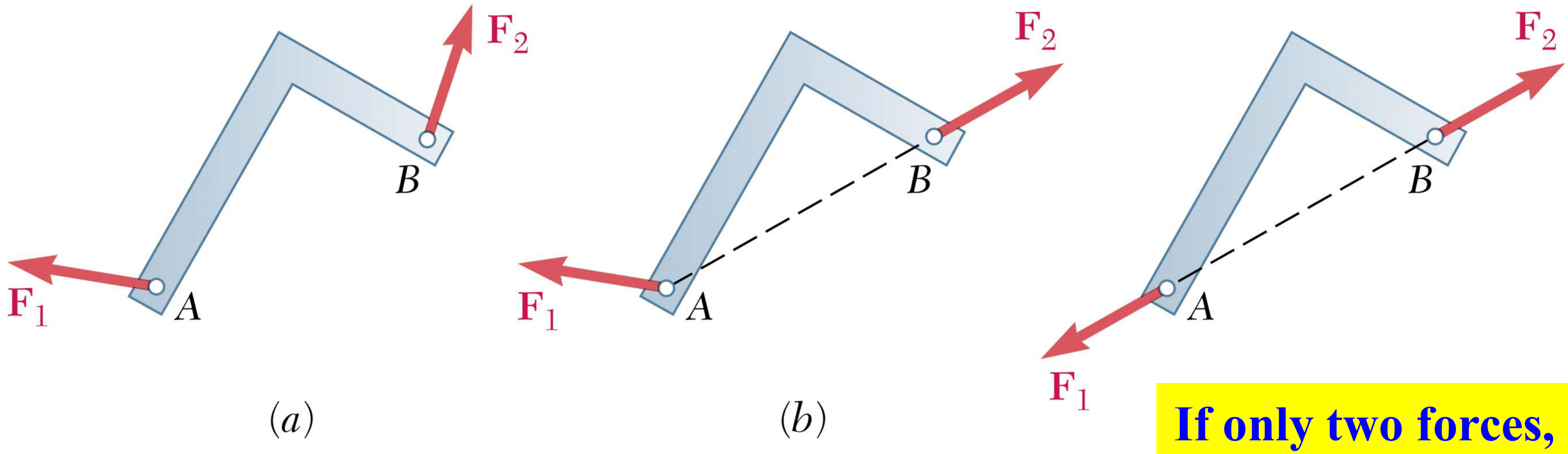
What if…?



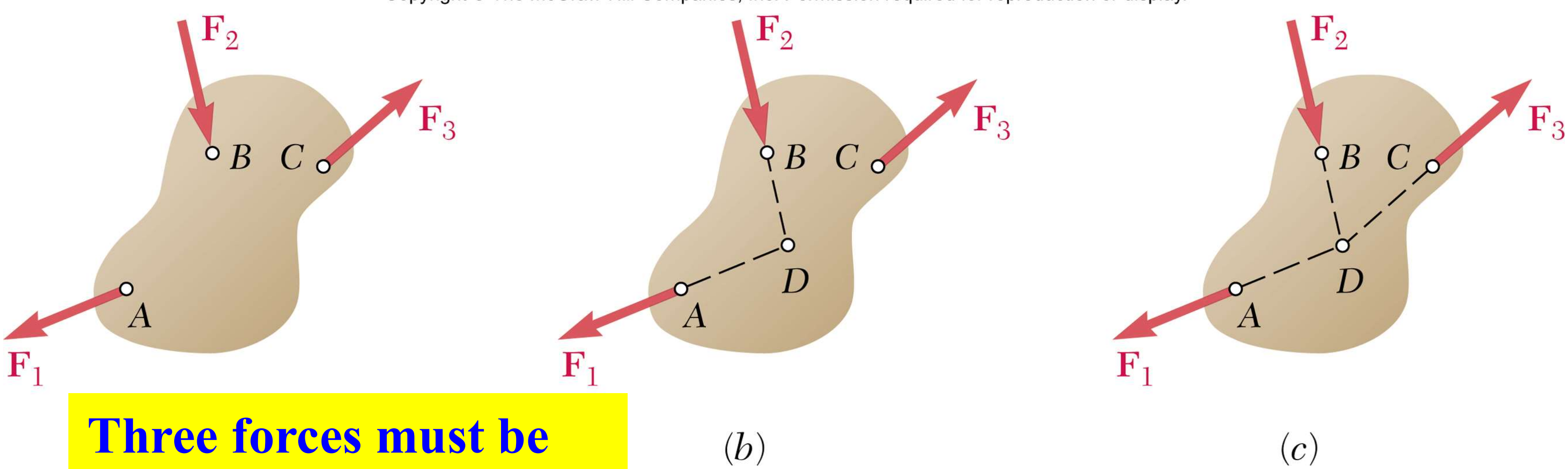
- Now suppose we have a different problem: How much gravel can this tractor carry before it tips over?



Equilibrium of a Two- or Three-Force Body

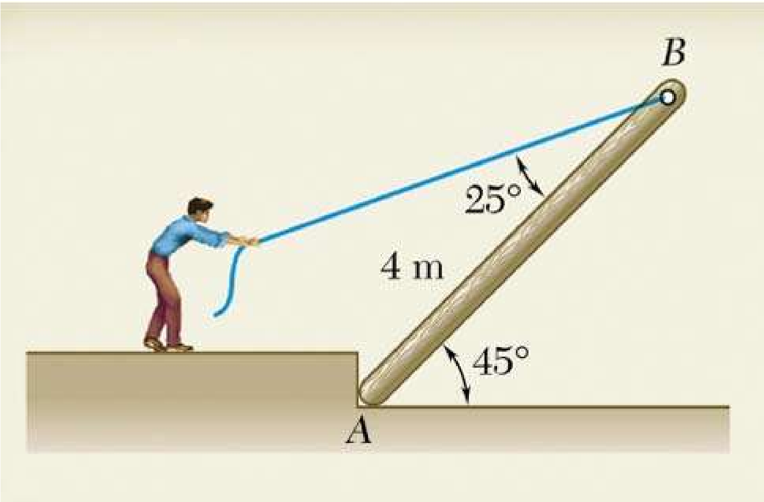


If only two forces, they must be in line!

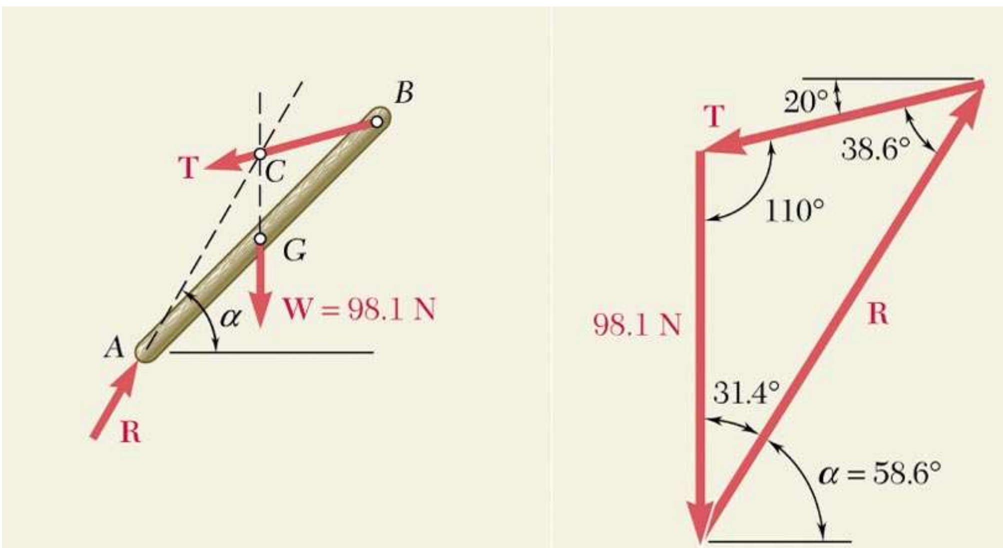


Three forces must be concurrent or parallel!

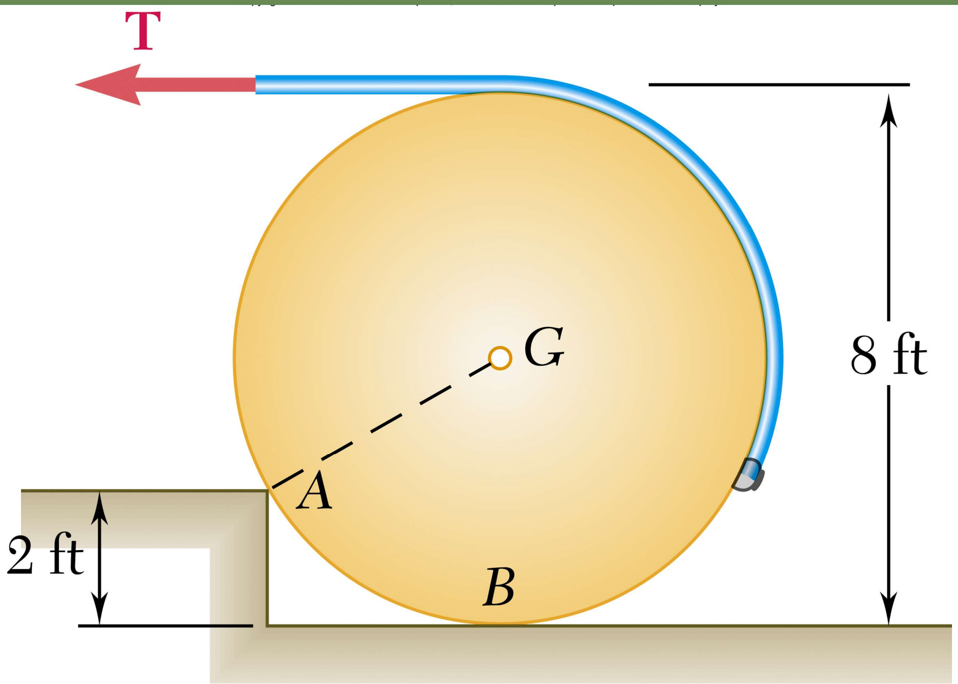
Sample Problem 4.6



A man raises a 10-kg joist, of length 4 m, by pulling on a rope. Find the tension *T* in the rope and the reaction at *A*.

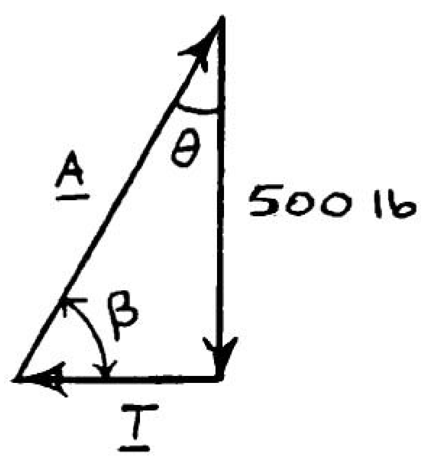


Problem 4.64



A 500-lb cylindrical tank, 8 ft in diameter, is to be raised over a 2-ft obstruction. A cable is wrapped around the tank and pulled horizontally as shown. Knowing that the corner of the obstruction at *A* is rough, find the required tension in the cable and the reaction at *A*.

Force triangle



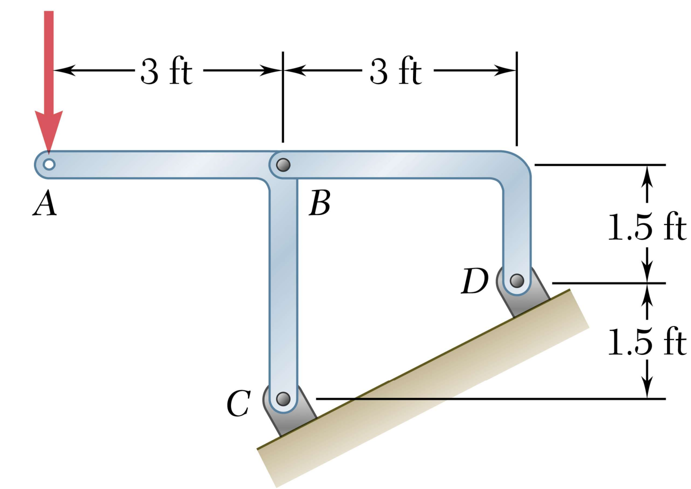
$$\cos\alpha = \frac{GD}{AG} = \frac{2\text{ ft}}{4\text{ ft}} = 0.5 \qquad \alpha = 60°$$

$$\theta = \frac{1}{2}\alpha = 30° \qquad (\beta = 60°)$$

$$T = (500\text{ lb})\tan 30° \qquad T = 289\text{ lb}$$

$$A = \frac{500\text{ lb}}{\cos 30°} \qquad \mathbf{A} = 577\text{ lb} \measuredangle\, 60.0°$$

Problem 4.66

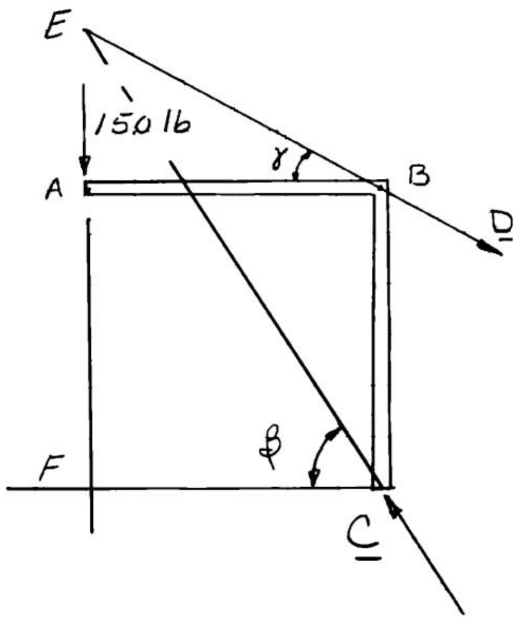


For the frame and loading shown, determine the reactions at *C* and *D*.

Since *BD* is a two-force member, the reaction at *D* must pass through Points *B* and *D*.

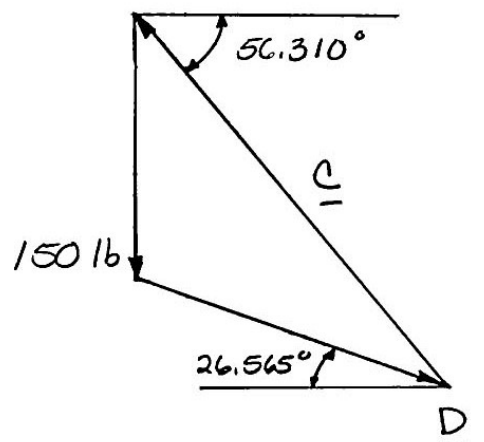
Free-Body Diagram:

(Three-force body)



$$\tan\beta = \frac{4.5\text{ ft}}{3\text{ ft}} \qquad \beta = 56.310°$$

$$\tan\gamma = \frac{1}{2} \qquad \gamma = 26.565°$$



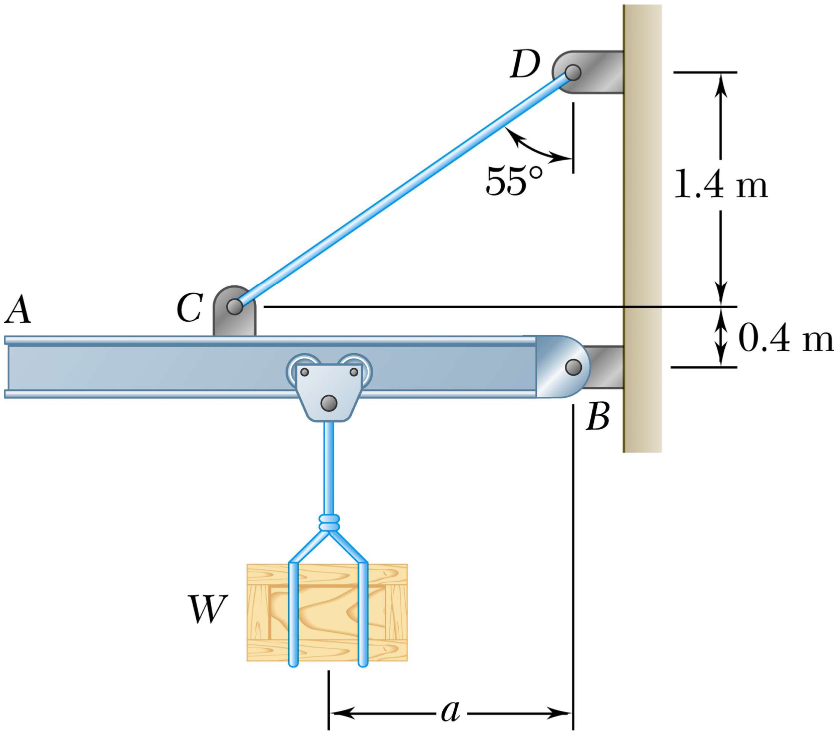
$$\frac{150\text{ lb}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{D}{\sin 33.690°}$$

$$C = 270.42\text{ lb},$$

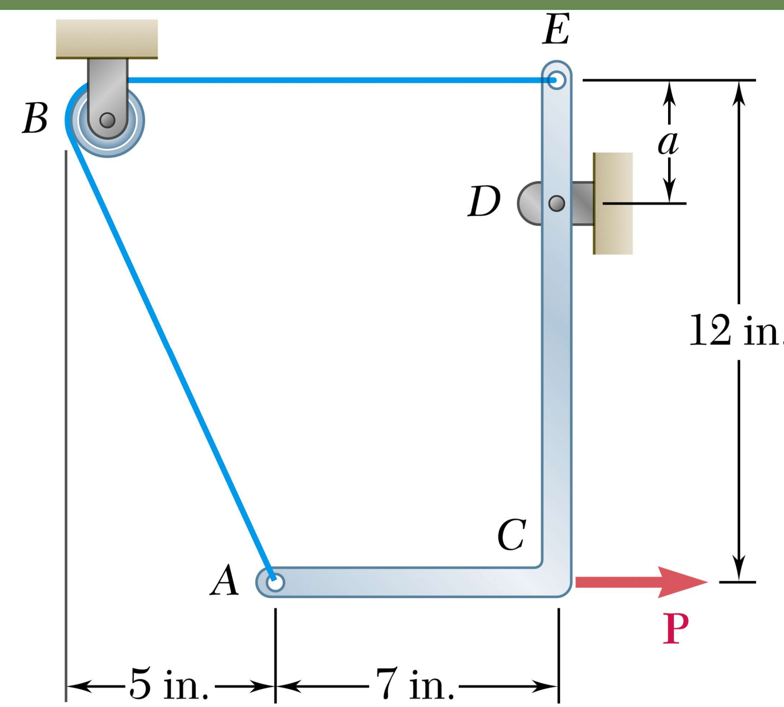
$$D = 167.704\text{ lb}$$

C = 270 lb ⦬ 56.3°; **D** = 167.7 lb ⦪ 26.6° ◀

Problem 4.73 4.29



A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5\ m$, determine (a) the tension in cable *CD*, (b) the reaction at *B*.



A force **P** of magnitude 90 lb is applied to member *ACE*, which is supported by a frictionless pin at *D* and by the cable *ABE*. Since the cable passes over a small pulley at *B*, the tension may be assumed to be the same in portions *AB* and *BE* of the cable. For the case when $a = 3$ in, determine (a) the tension in the cable, (b) the reaction at D.

Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

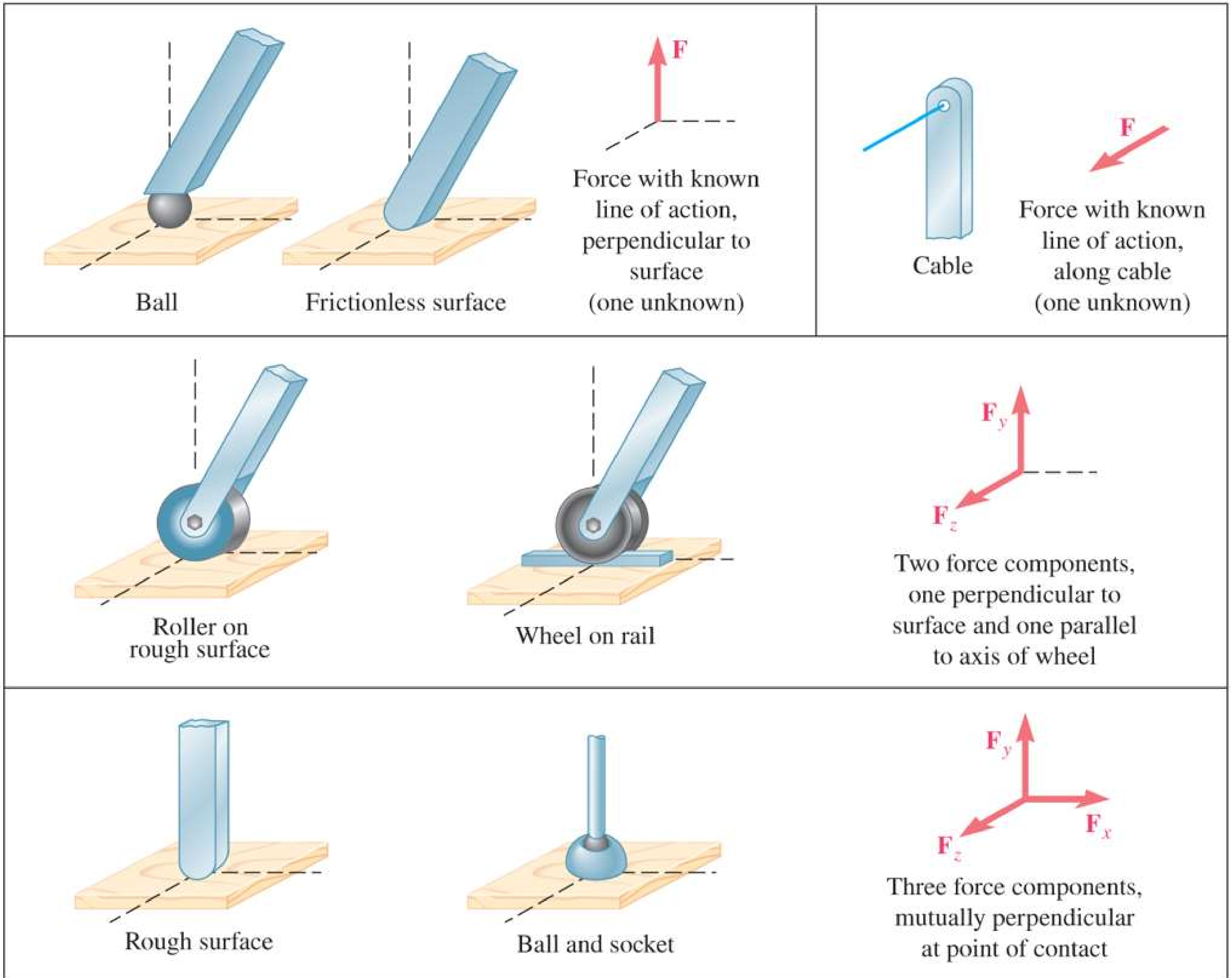
$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections or unknown applied forces.

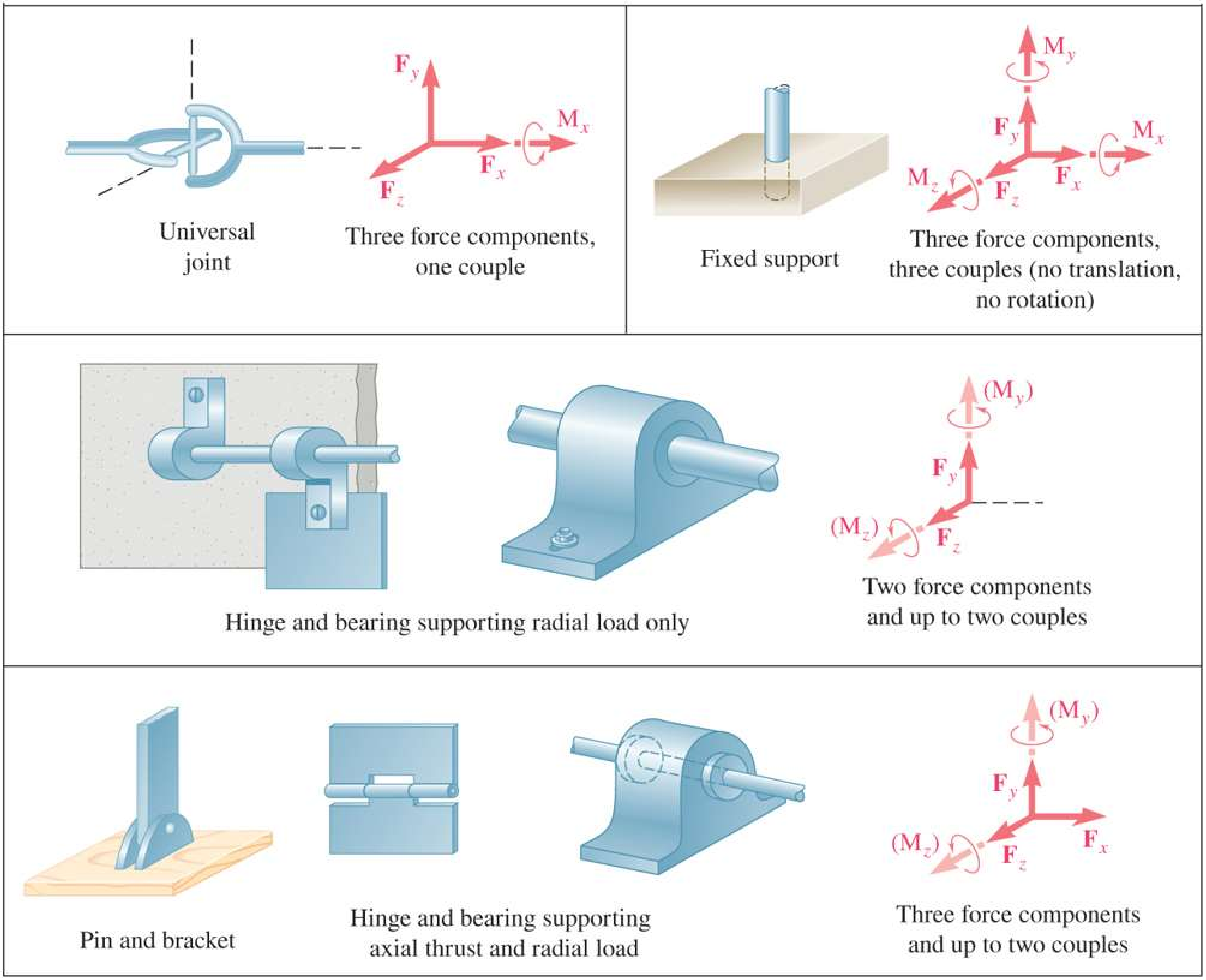
- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

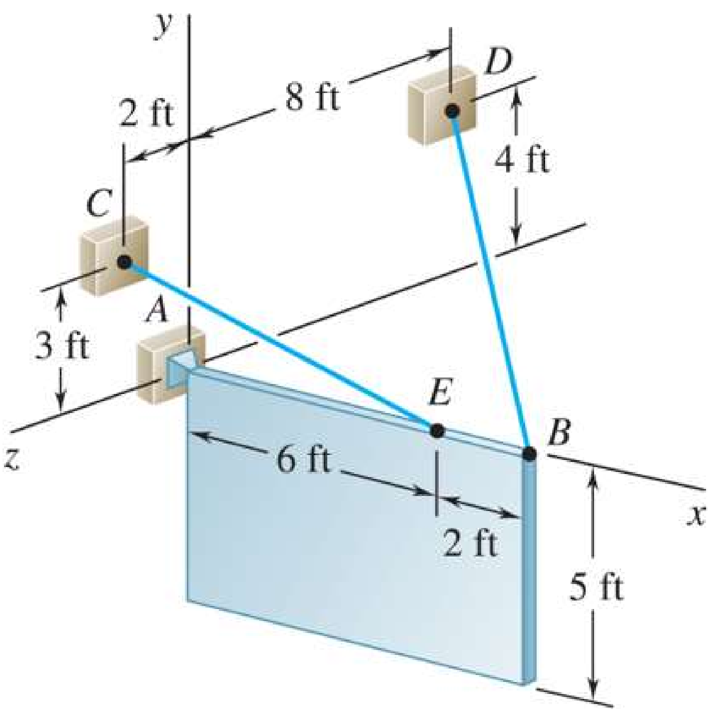
Reactions at Supports and Connections for a Three-Dimensional Structure

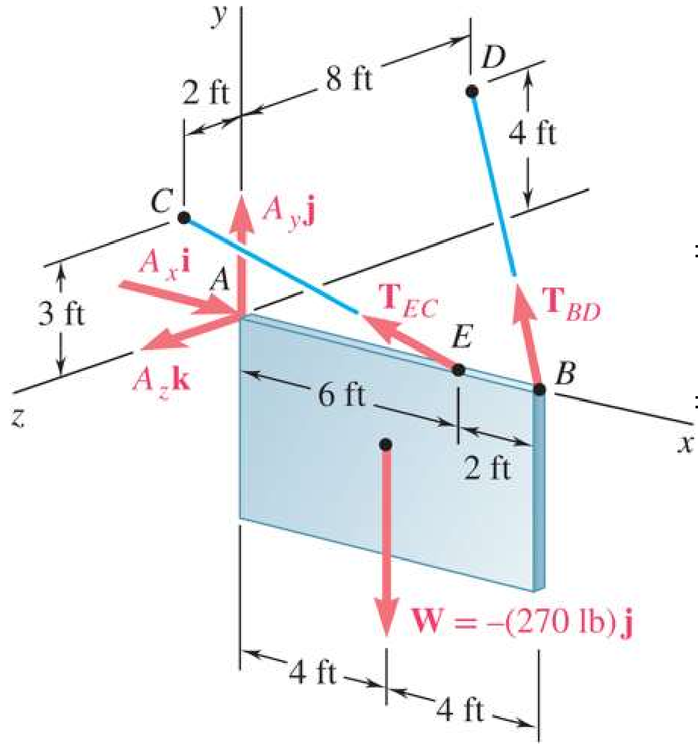


Reactions at Supports and Connections for a Three-Dimensional Structure



Sample Problem 4.8





$$= T_{BD}\frac{-8\vec{i}+4\vec{j}-8\vec{k}}{12}$$

$$= T_{EC}\frac{-6\vec{i}+3\vec{j}+2\vec{k}}{7}$$

A sign of uniform density weighs 270 lb and is supported by a ball-and-socket joint at *A* and by two cables.

Determine the tension in each cable and the reaction at *A*.

$$\sum \vec{M}_A = \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{EC} + (4\ \text{ft})\vec{i} \times (-270\ \text{lb})\vec{j} = 0$$

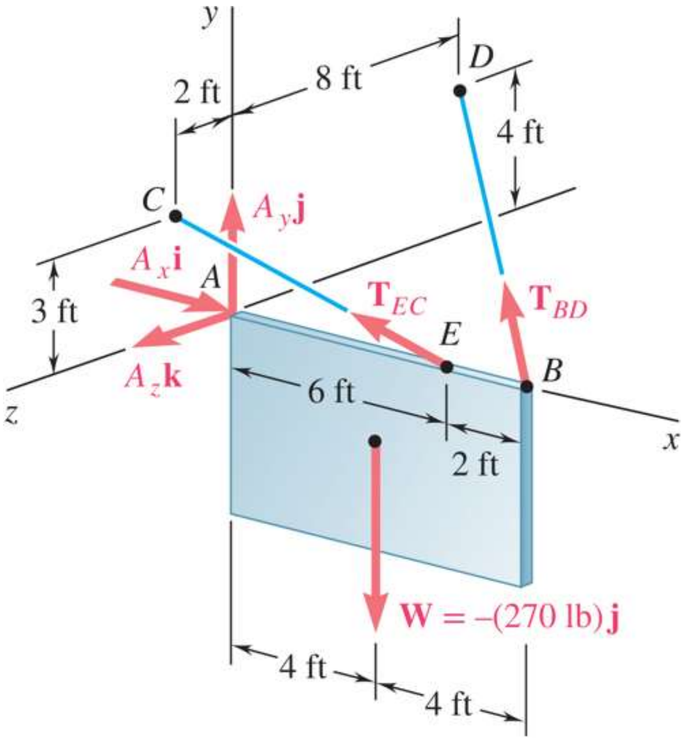
$$\vec{j}: \quad 5.333\,T_{BD} - 1.714\,T_{EC} = 0$$

$$\vec{k}: \quad 2.667\,T_{BD} + 2.571\,T_{EC} - 1080\,\text{lb} = 0$$

$$T_{BD} = 101.3\ \text{lb} \qquad T_{EC} = 315\ \text{lb}$$

$$\vec{A} = (338\ \text{lb})\vec{i} + (101.2\ \text{lb})\vec{j} - (22.5\ \text{lb})\vec{k}$$

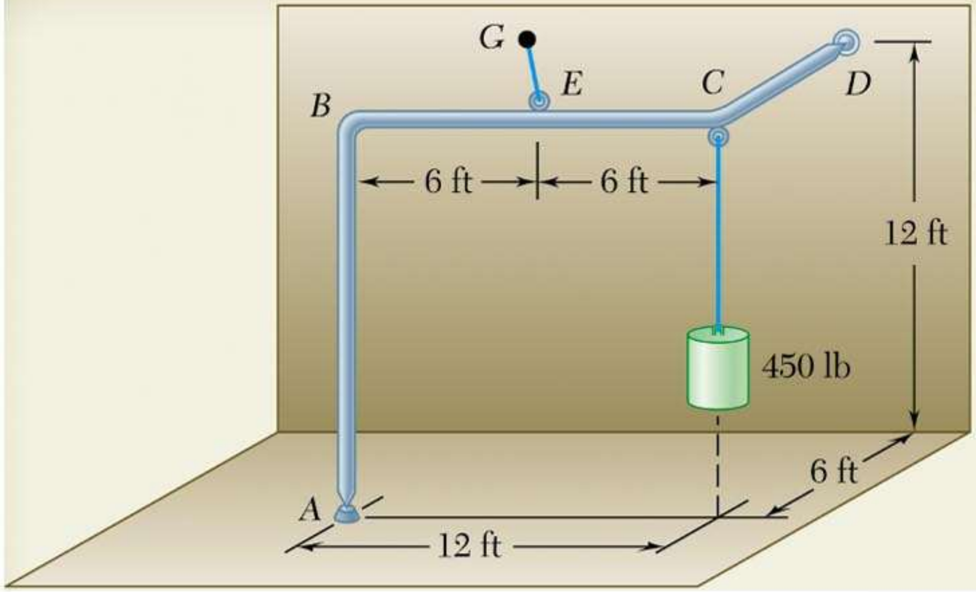
What if…?



Could this sign be in static equilibrium if cable BD were removed?

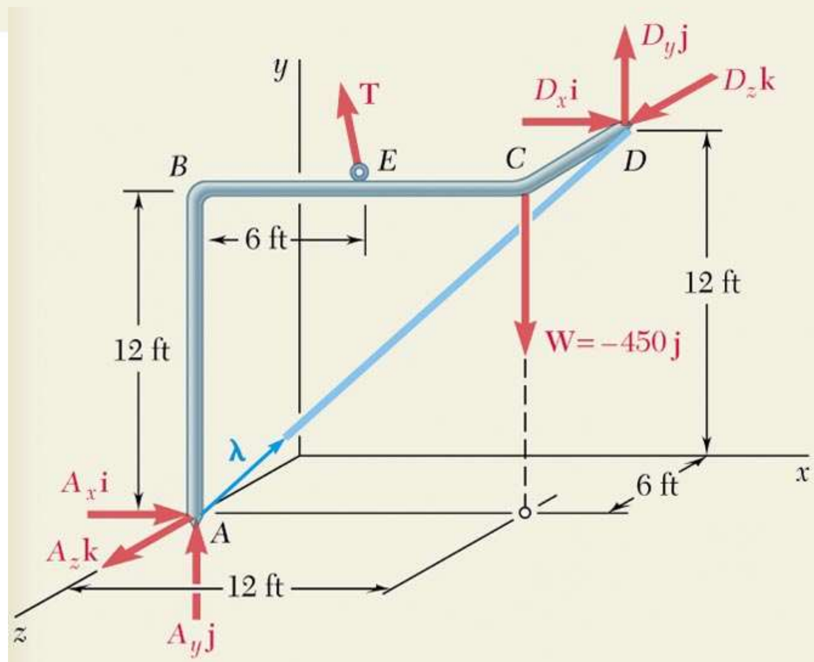
The sign could not be in static equilibrium because T_{EC} causes a moment about the *y* axis (due to the existence of T_{ECz}) that must be countered by an equal and opposite moment. This can only be provided by a cable tension that has a *z* component in the negative *z* direction, such as what T_{BD} provides.

Sample Problem 4.10



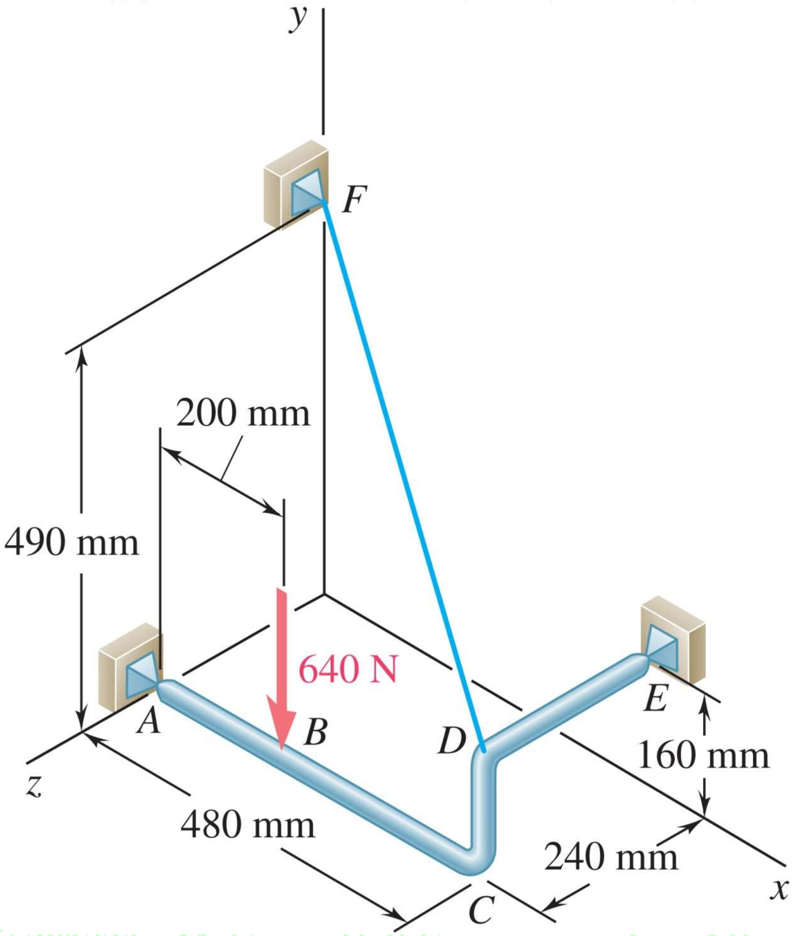
SAMPLE PROBLEM 4.10

A 450-lb load hangs from the corner C of a rigid piece of pipe ABCD which has been bent as shown. The pipe is supported by the ball-and-socket joints A and D, which are fastened, respectively, to the floor and to a vertical wall, and by a cable attached at the midpoint E of the portion BC of the pipe and at a point G on the wall. Determine (a) where G should be located if the tension in the cable is to be minimum, (b) the corresponding minimum value of the tension.



Prob. 4.138

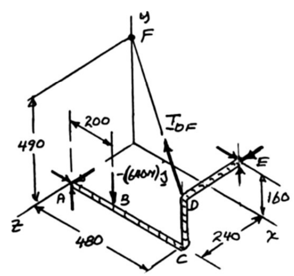
The pipe ACDE is supported by ball-and-socket joints at A and E and by the wire DF. Determine the tension in the wire when a 640-N load is applied at B as shown.



SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overrightarrow{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \qquad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = T_{DF}\frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF}\frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

$$\Sigma M_{AE} = \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

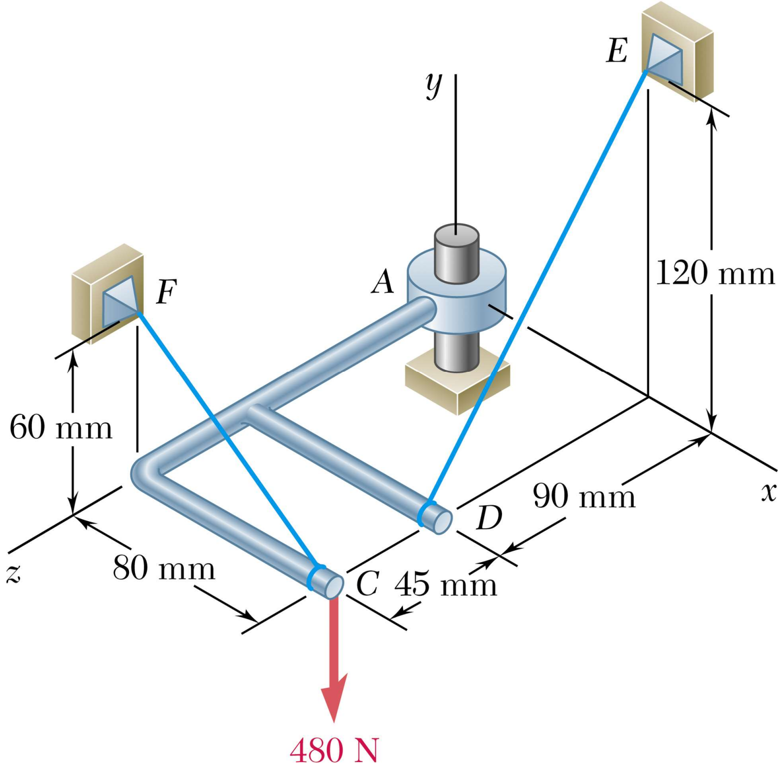
$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

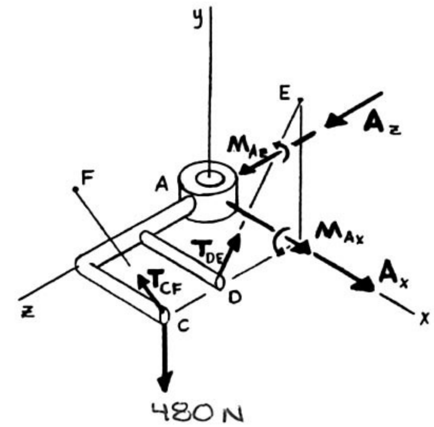
$$-1120 T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.86 \text{ N} \qquad T_{DF} = 343 \text{ N} \blacktriangleleft$$

Problem 4.121



The assembly shown is welded to collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.



$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2} \text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12 \text{ m})\mathbf{j} - (0.09 \text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2} \text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480 \text{ N} = 0$$

$$0.6T_{CF} + 0.8T_{DE} = 480 \text{ N}$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135 \text{ m}) + (0.6T_{DE})(0.08 \text{ m}) = 0$$

$$T_{DE} = 2.25T_{CF}$$

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480 \text{ N}$$

$$T_{CF} = 200.00 \text{ N} \quad ◀$$

$$T_{DE} = 2.25(200.00 \text{ N}) = 450.00 \text{ N} \quad ◀$$

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00 \text{ N}) = 0 \qquad A_z = 270.00 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00 \text{ N}) = 0 \qquad A_x = 160.000 \text{ N}$$

$$\mathbf{A} = (160.0 \text{ N})\mathbf{i} + (270 \text{ N})\mathbf{k} \quad ◀$$

$$\Sigma M_x = 0: \quad M_{A_x} + (480 \text{ N})(0.135 \text{ m}) - [(200.00 \text{ N})(0.6)](0.135 \text{ m}) - [(450 \text{ N})(0.8)](0.09 \text{ m}) = 0$$

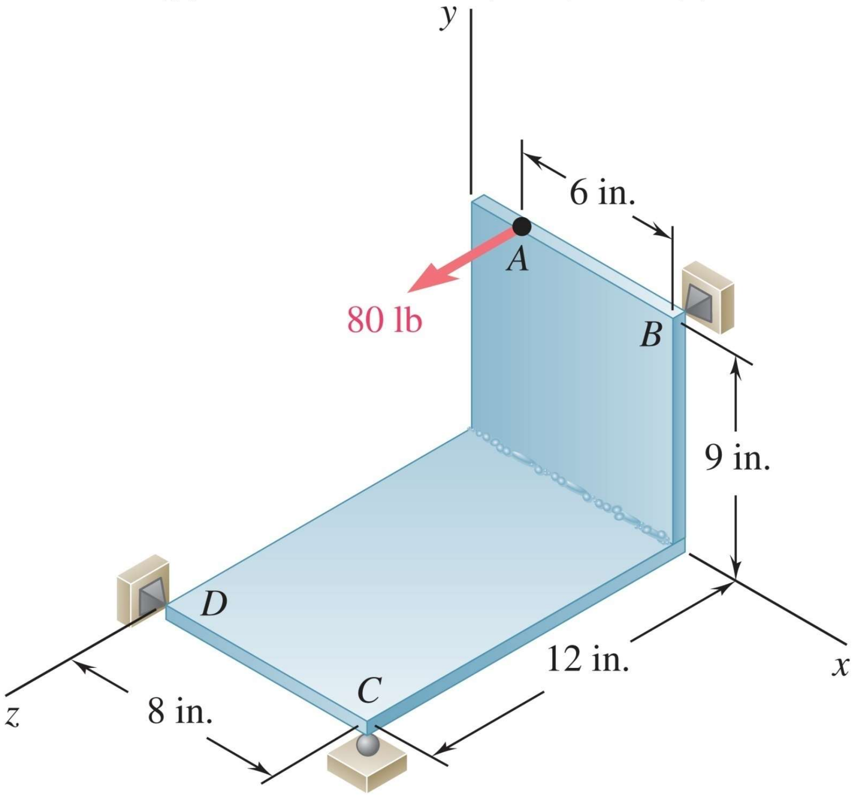
$$M_{A_x} = -16.2000 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480 \text{ N})(0.08 \text{ m}) + [(200.00 \text{ N})(0.6)](0.08 \text{ m}) + [(450 \text{ N})(0.8)](0.08 \text{ m}) = 0$$

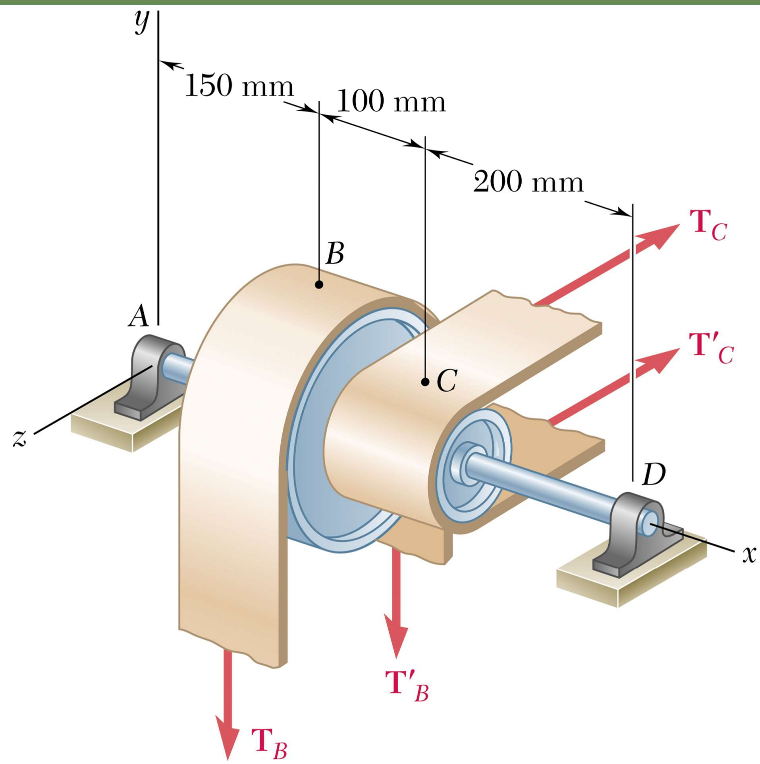
$$M_{A_z} = 0 \qquad \mathbf{M}_A = -(16.20 \text{ N}\cdot\text{m})\mathbf{i} \quad ◀$$

Prob. 4.137

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and socket joints at B and D and by a ball on a horizontal surface at C. For the loading shown, determine the reaction at C.



Problem 4.95



Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at *A* and *D*. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt *B* and 150 N in both portions of belt *C*, determine the reactions at *A* and *D*. Assume that the bearing at *D* does not exert any axial thrust.