$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A} \qquad R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$\vec{R} = \vec{P} + \vec{Q}$$

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$F_x = F_h \cos\phi$$
$$= F\sin\theta_y \cos\phi$$
$$F_y = F_h \sin\phi$$
$$= F\sin\theta_y \sin\phi$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$
$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$
$$= F\vec{\lambda}$$
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

$$\vec{P}\bullet\vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

$$\boldsymbol{M_O} = \boldsymbol{r}\times\boldsymbol{F} \qquad \vec{P}\bullet\vec{P} = P_x^2 + P_y^2 + P_z^2 = P^2$$

$$M_x = yF_z - zF_y$$
$$M_y = zF_x - xF_z$$
$$M_z = xF_y - yF_x$$

$$\vec{M}_{O'} = \vec{r}\,'\times\vec{F} = \left(\vec{r} + \vec{s}\right)\times\vec{F} = \vec{r}\times\vec{F} + \vec{s}\times\vec{F}$$
$$= \vec{M}_O + \vec{s}\times\vec{F}$$

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

$$\vec{P}\times\vec{Q} = \left(P_yQ_z - P_zQ_y\right)\vec{i} + \left(P_zQ_x - P_xQ_z\right)\vec{j} + \left(P_xQ_y - P_yQ_x\right)\vec{k} \quad = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{pmatrix}$$

$$\vec{S}\bullet(\vec{P}\times\vec{Q}) = S_x\left(P_yQ_z - P_zQ_y\right) + S_y\left(P_zQ_x - P_xQ_z\right) + S_z\left(P_xQ_y - P_yQ_x\right)$$
$$= \begin{pmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{pmatrix}$$

$\sum M_y \quad \bar{x}W = \sum x\Delta W = \int x\,dW \quad \sum M_y \quad \bar{y}W = \sum y\Delta W = \int y\,dW$

For lines: $\bar{x}\left(\gamma La\right) = \int x\left(\gamma a\right)dL \quad \bar{x}L = \int x\,dL \quad \bar{y}L = \int y\,dL$

Centroid: $\bar{x}A = \int x dA = \iint x\,dx dy = \int \bar{x}_{el}\,dA \quad \bar{y}A = \int y dA = \iint y\,dx dy = \int \bar{y}_{el}\,dA$

Pappas-Guldinus $\quad A = 2\pi \bar{y}L \qquad V = 2\pi \bar{y} A$

Moments of Inertia $I_x = \int y^2 dA \qquad I_y = \int x^2 dA \qquad J_0 = \int r^2 dA = I_y + I_x$

radius of gyration $\quad I_x = k_x^2 A \quad I_y = k_y^2 A \quad J_O = k_O^2 A \quad$ Parallel axis theorem: $I = \bar{I} + Ad^2$

Product of inertia $I_{xy} = \int xy\,dA \qquad I_{xy} = \bar{I}_{xy} + \bar{x}\bar{y}A$

$$x' = x\cos\theta + y\sin\theta \quad y' = y\cos\theta - x\sin\theta \quad I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta \quad I_{x'y'} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta \quad \left(I_{x'} - I_{ave}\right)^2 + I_{x'y'}^2 = R^2$$

$$I_{ave} = \frac{I_x + I_y}{2} \quad R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \quad I_{\max,\min} = I_{ave} \pm R \quad \tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$r^2\Delta m$ = *moment of inertia* of the mass $\Delta m \quad I = k^2 m \quad k = \sqrt{I/m} \quad I_y = \int r^2 dm = \int\left(z^2 + x^2\right)\,dm$

$I_x = \int\left(y^2 + z^2\right)dm \quad I_z = \int\left(x^2 + y^2\right)dm$ *Parallel axis theorem* $I_x = \bar{I}_{x'} + m\left(\bar{y}^2 + \bar{z}^2\right)$

$I_y = \bar{I}_{y'} + m\left(\bar{z}^2 + \bar{x}^2\right) \quad I_z = \bar{I}_{z'} + m\left(\bar{x}^2 + \bar{y}^2\right) \quad I = \bar{I} + md^2$

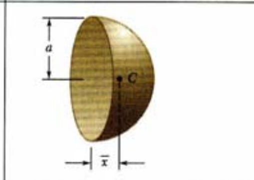
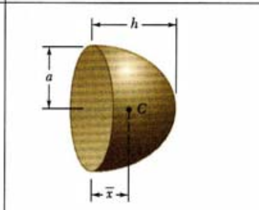
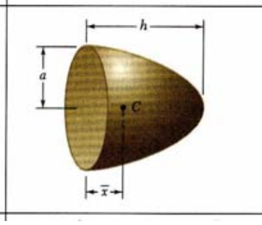
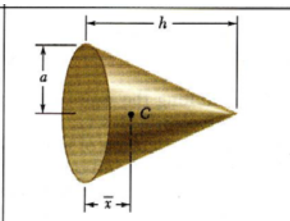
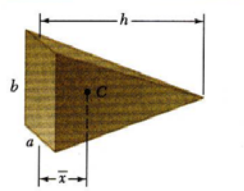
$$\sum \vec{F}_i = \sum m_i\vec{a}_i \quad \sum\left(\vec{r}_i \times \vec{F}_i\right) = \sum\left(\vec{r}_i \times m_i\vec{a}_i\right) \quad \vec{L} = \sum_{i=1}^{n} m_i\vec{v}_i \quad \dot{\vec{L}} = \sum_{i=1}^{n} m_i\dot{\vec{v}}_i = \sum_{i=1}^{n} m_i\vec{a}_i \quad \sum \vec{F} = \dot{\vec{L}}$$

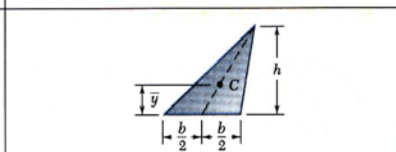
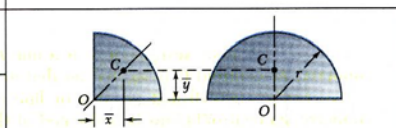
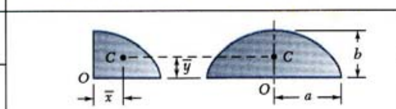
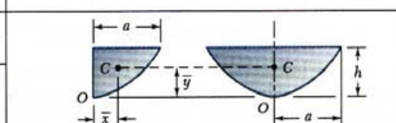
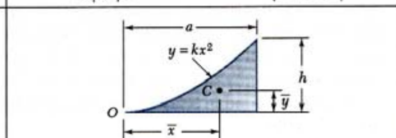
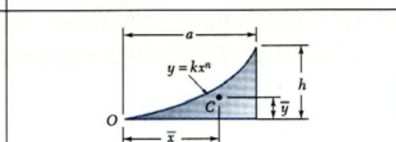
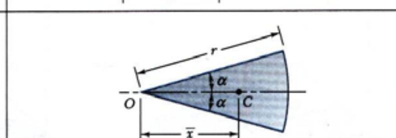
$$\vec{H}_O = \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{v}_i\right) \quad \dot{\vec{H}}_O = \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{a}_i\right) \quad \sum \vec{M}_O = \dot{\vec{H}}_O$$

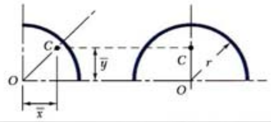
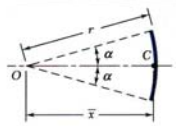
Center of Mass: $m\vec{r}_G = \sum_{i=1}^{n} m_i\vec{r}_i \quad m\vec{v}_G = \sum_{i=1}^{n} m_i\vec{v}_i = \vec{L} \quad m\vec{a}_G = \dot{\vec{L}} = \sum \vec{F} \quad \vec{H}'_G = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{v}'_i\right)$

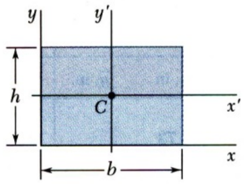
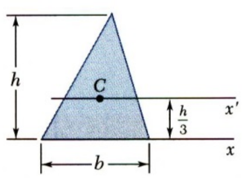
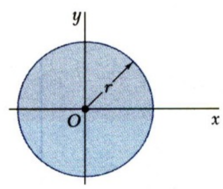
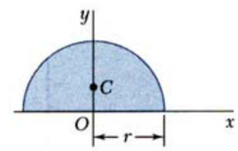
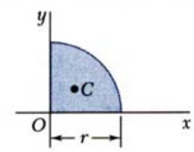
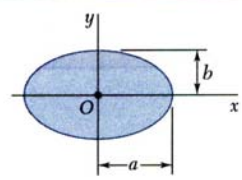
$$\dot{\vec{H}}'_G = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{a}'_i\right) = \sum_{i=1}^{n}\left(\vec{r}'_i \times \vec{F}_i\right) = \sum \vec{M}_G$$

$$T = \tfrac{1}{2}\sum_{i=1}^{n} m_i\left(\vec{v}_i \bullet \vec{v}_i\right) = \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i^2 \quad T = \tfrac{1}{2}m\bar{v}_G^2 + \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i'^2 \quad \vec{L}_1 + \sum\int_{t_1}^{t_2}\vec{F}dt = \vec{L}_2 \quad \vec{H}_1 + \sum\int_{t_1}^{t_2}\vec{M}_O dt = \vec{H}_2$$

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Rectangle		$\bar{I}_{x'} = \frac{1}{12} bh^3$ $\bar{I}_{y'} = \frac{1}{12} b^3 h$ $I_x = \frac{1}{3} bh^3$ $I_y = \frac{1}{3} b^3 h$ $J_C = \frac{1}{12} bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36} bh^3$ $I_x = \frac{1}{12} bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4} \pi r^4$ $J_O = \frac{1}{2} \pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8} \pi r^4$ $J_O = \frac{1}{4} \pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16} \pi r^4$ $J_O = \frac{1}{8} \pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4} \pi ab^3$ $\bar{I}_y = \frac{1}{4} \pi a^3 b$ $J_O = \frac{1}{4} \pi ab(a^2 + b^2)$

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \dot{\vec{r}}_B = \dot{\vec{r}}_A + \dot{\vec{r}}_{B/A} \qquad \ddot{\vec{r}}_B = \ddot{\vec{r}}_A + \ddot{\vec{r}}_{B/A}$$

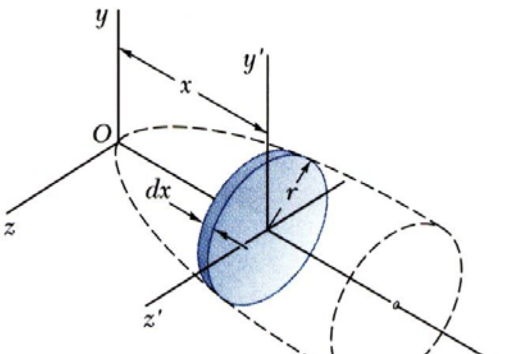
$$\vec{v} = \frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r} \qquad \vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r} = \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r} \qquad \vec{a}_t = \alpha \vec{k} \times \vec{r} \quad \vec{a}_n = -\omega^2 \vec{r}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta} \qquad \begin{aligned} \omega &= \omega_0 + \alpha t \\ \theta &= \theta_0 + \omega_0 t + \tfrac{1}{2} \alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0) \end{aligned}$$

rigid body $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{v}_{B/A} = \omega \vec{k} \times \vec{r}_{B/A} \qquad v_{B/A} = r\omega \qquad \vec{v}_B = \vec{v}_A + \omega \vec{k} \times \vec{r}_{B/A}$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \left(\vec{a}_{B/A}\right)_t = \alpha \vec{k} \times \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_t = r\alpha \qquad \left(\vec{a}_{B/A}\right)_n = -\omega^2 \vec{r}_{B/A} \qquad \left(a_{B/A}\right)_n = r\omega^2$$

$$\sum F_x = m\bar{a}_x \qquad \sum F_y = m\bar{a}_y \qquad \sum M_G = \bar{I} \alpha$$

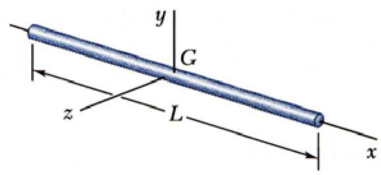


$dm = \rho \pi r^2 \, dx$

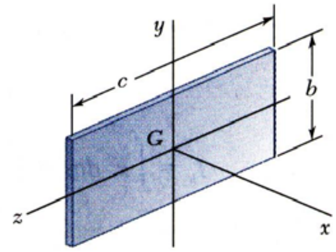
$dI_x = \frac{1}{2} r^2 \, dm$

$dI_y = dI_{y'} + x^2 \, dm = \left(\frac{1}{4} r^2 + x^2\right) dm$

$dI_z = dI_{z'} + x^2 \, dm = \left(\frac{1}{4} r^2 + x^2\right) dm$



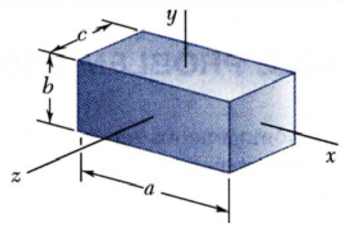
$I_y = I_z = \frac{1}{12} m L^2$



$I_x = \frac{1}{12} m(b^2 + c^2)$

$I_y = \frac{1}{12} m c^2$

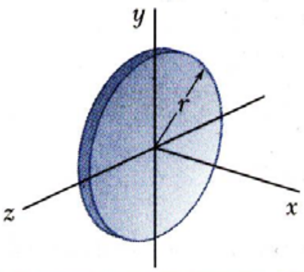
$I_z = \frac{1}{12} m b^2$



$I_x = \frac{1}{12} m(b^2 + c^2)$

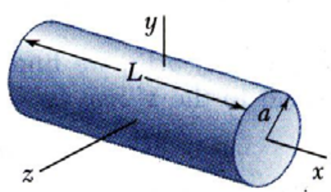
$I_y = \frac{1}{12} m(c^2 + a^2)$

$I_z = \frac{1}{12} m(a^2 + b^2)$



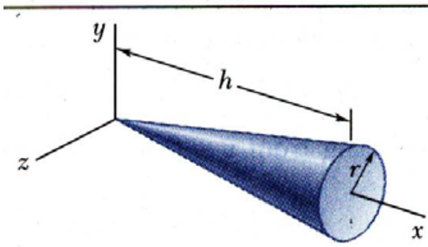
$I_x = \frac{1}{2} m r^2$

$I_y = I_z = \frac{1}{4} m r^2$



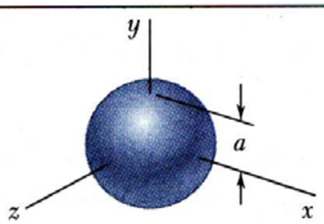
$I_x = \frac{1}{2} m a^2$

$I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$



$I_x = \frac{3}{10} m a^2$

$I_y = I_z = \frac{3}{5} m(\frac{1}{4} a^2 + h^2)$



$I_x = I_y = I_z = \frac{2}{5} m a^2$