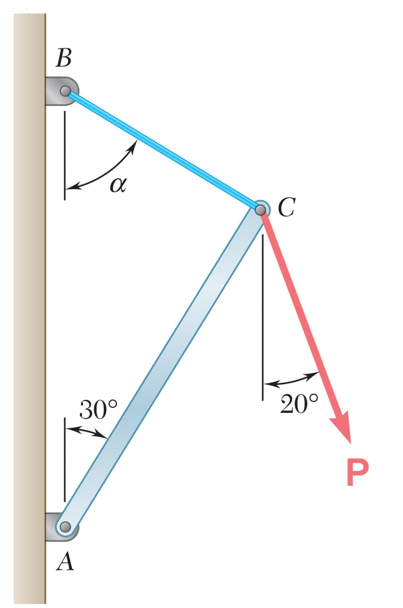
Phys 3200 Fall 2024 Exam #1 Name:____

You may use your non-programmable calculator. You may not consult books or notes. You must show your work. **Box or circle your answers**. No partial credit is given unless so indicated.

1. (20 points) Knowing that α = 50º, determine the force P such that the tension in the cable is 300 lb.



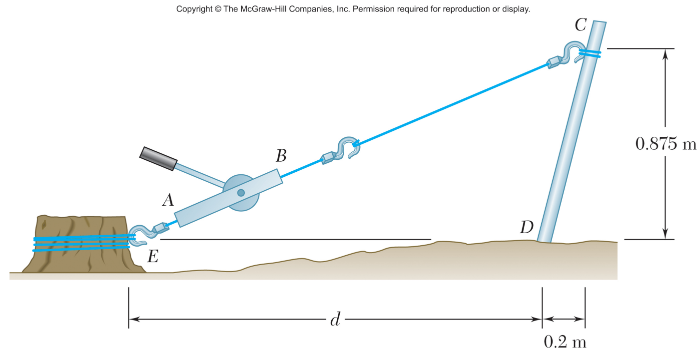
Solution:

Angles are 50º, 30º, and 100º.

Law of Sines:

$$\frac{300}{\sin 50^o} = \frac{P}{\sin 100^o} \qquad \boxed{P = 386\, lb}$$

2. (20 points, Partial Credit) It is known that a force with a moment of 960 *N*m* about *D* is required to straighten the fence post *CD*. If the capacity of winch puller *AB* is 2800 N, determine the minimum value of distance *AC* to create the specified moment about point *D*.



$$CD = \sqrt{0.875^2 + 0.2^2} = 0.8976m$$

Lever arm required $= \dfrac{960}{2800} = 0.343m$

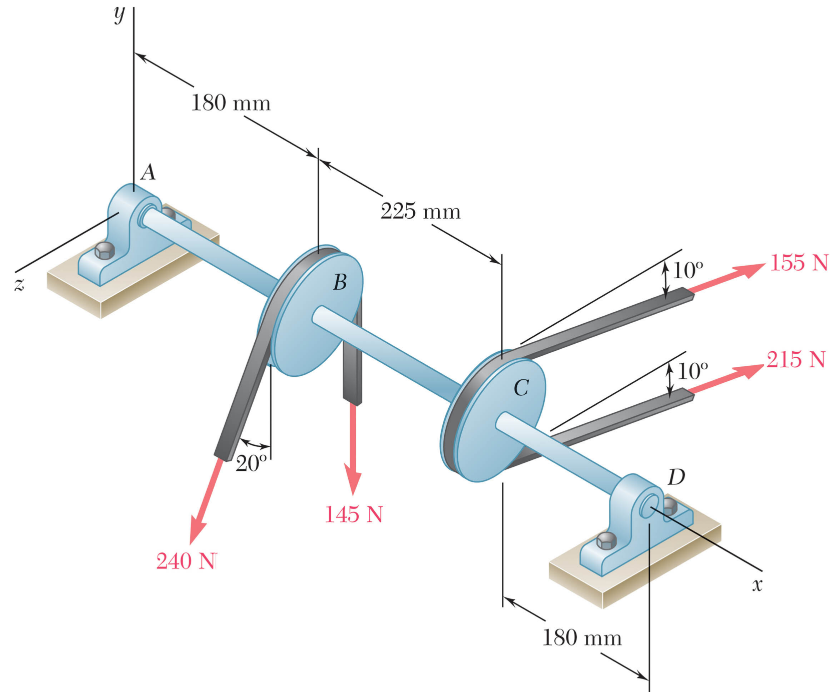
Angle at C is $= \sin^{-1}\left(\dfrac{0.343}{0.8976}\right) = 22.5^o$

Angle at D is $= \pi - \tan^{-1}\left(\dfrac{0.875}{0.2}\right) = 102.9^o$

Angle at A is $= \pi - 102.9^o - 22.5^o = 54.6^o$

Law of Sines: $AC = \sin 102.9^o \times \dfrac{0.8976}{\sin 54.6^o} = \boxed{1.073m}$

3. (20 points, Partial Credit) Two 150-mm-diameter pulleys are mounted on line shaft *AD*. The belts at *B* and *C* lie in vertical planes parallel to the *yz* plane. Replace the belt forces shown with an equivalent force-couple system at *A*.



Solution:

The two forces on belt at C combine to a total force of

$\vec{F}_C = 370N \cdot (-\sin 10^o \vec{j} - \cos 10^o \vec{k})$

$= (-64.2\vec{j} - 364\vec{k})\ N$

and a moment about C of

$\vec{M}_C = 75 \cdot 60\vec{i}\ \ mm \cdot N$.

When tranferred to point A, the moment of these two forces is

$\vec{M}_{AC} = 4500\vec{i} + 405 \cdot 370(\cos 10^o \vec{j} - \sin 10^o \vec{k})\ \ mm \cdot N$

$= (4.50\vec{i} + 147.6\vec{j} - 26.0\vec{k})\ \ m \cdot N$

The two forces on belt at B combine to a total of

$\vec{F}_B = [(-240\cos 20^o - 145)\vec{j}\ + 240\sin 20^o \vec{k}]N\ = (-370\vec{j} + 82.1\vec{k})N$

and a moment about B of

$\vec{M}_B = 75 \cdot 95\vec{i}\ \ mm \cdot N = 7.12\vec{i}\ \ m \cdot N$.

When tranferred to point A, the couple of these two forces is

$\vec{M}_{AB} = (7.12\vec{i} - 14.78\vec{j} - 66.7\vec{k})\ \ m \cdot N$

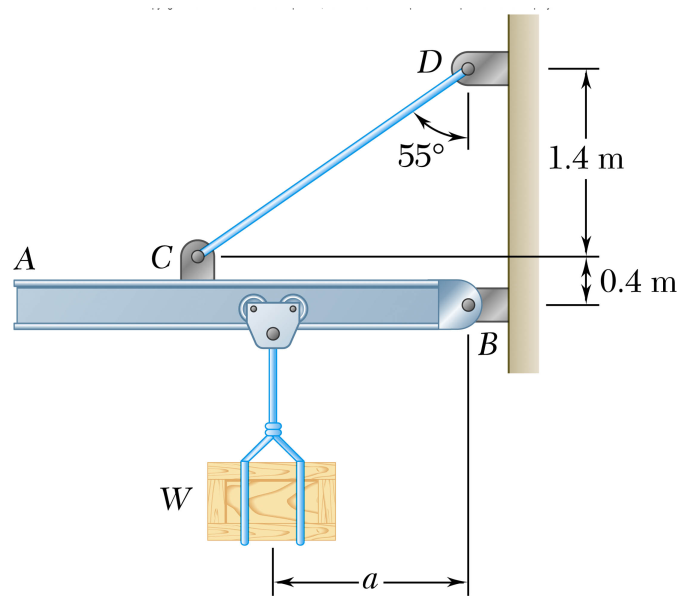
The total force at point A is

$\vec{F}_A = \vec{F}_B + \vec{F}_C = (-434\vec{j} - 282\vec{k})N$

and the total couple at A is

$\vec{M}_A = (4.50\vec{i} + 147.6\vec{j} - 26.0\vec{k}\ + 7.12\vec{i} - 14.78\vec{j} - 66.7\vec{k})\ \ m \cdot N = (11.62\vec{i} + 132.8\vec{j} - 92.7\vec{k})\ \ m \cdot N$

4. (20 points) A 50-kg crate is attached to the trolley-beam system shown. Knowing that *a* = 1.2 *m*, determine (a) the tension in cable *CD*, (b) the reaction at *B*.



Solution:

Three forces acting on the massless steel beam sum to zero force and zero moment. The moment about B vanishes, from which we write

$$M_B = 0 = -T \cdot 1.8 \sin 55^o + 50 * 9.8 \cdot 1.2 \ (m \cdot N)$$

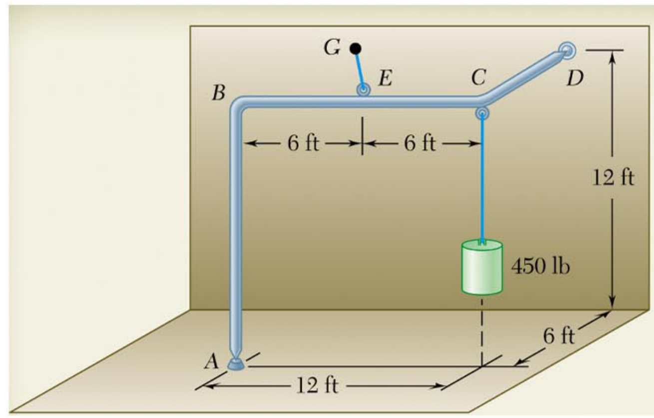
$$\boxed{T = 399N}$$

$0 = B_X + 399N \sin 55^o$ $\quad$ $\boxed{B_X = -327N}$

$0 = B_y - 50 * 9.8 + 399 \cos 55^o$ $\quad$ $\boxed{B_y = 261N}$

Or, $B = \sqrt{327^2 + 261^2} = \boxed{419N \quad \text{at an angle of } 180^o - \tan^{-1}(261/327) = 141.4^o}$

5. A 450-lb load hangs from the corner *C* of a rigid piece of pipe *ABCD* which has been bent as shown. The pipe is supported by the ball-and-socket joints *A* and *D*, which are fastened, resectively, to the floor and to a vertical wall and by a cable attached at the midpoint E of the portion *BC* of the pipe to a point *G* directly cross on the wall, such that cable EG is parallel to the pipe section CD. **(10 points)** What is the tension T in the cable? **(10 points +10 Extra Credit)** What is the reaction at A?



Solution:

The total moment about the line AD, which has a direction unit vector of

$\vec{\lambda}_{AD} = \dfrac{12\vec{i} + 12\vec{j} - 6\vec{k}}{\sqrt{12^2 + 12^2 + 6^2}} = \dfrac{2}{3}\vec{i} + \dfrac{2}{3}\vec{j} - \dfrac{1}{3}\vec{k}$, should vanish.

$$0 = \left(\frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}\right) \cdot \left[(12\vec{i} + 12\vec{j}) \times (-450\vec{j}) + (6\vec{i} + 12\vec{j}) \times (-T\,\vec{k})\right] \qquad \boxed{T = 450\,lb}$$

Moment about C vanishes

$$0 = (-6\vec{i}) \times (-450\vec{k}) + (-6\vec{k}) \times (D_x\vec{i} + D_y\vec{j}) + (-12\vec{i} - 12\vec{j}) \times (A_x\vec{i} + A_y\vec{j} + A_z\vec{k})$$
$$= (D_y - 2A_z)\vec{i} + (-450 + 2A_z - D_x)\vec{j} + (2A_x - 2A_y)\vec{k}$$

Force vanishes

$$0 = (D_x + A_x)\vec{i} + (D_y + A_y - 450)\vec{j} + (D_z + A_z - 450)\vec{k}$$

Moment about D vanishes

$$0 = (-2A_z + A_y - 450)\vec{i} + (-A_x + 2A_z - 450)\vec{j} + (-2A_y + 2A_x)\vec{k}$$

$$\boxed{A_x = 450\,lb \quad A_y = 450\,lb \quad A_z = 0}$$

($D_x = -450; \quad D_y = 0; \quad D_z = 450$)