8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series - \( C_{eq} = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} \)

All three capacitors in parallel - \( C_{eq} = C_1 + C_2 + C_3 \)

One capacitor in series with a parallel combination of the other two:

\[ C_{eq} = \left( \frac{1}{C_1 + C_2} + \frac{1}{C_1} \right)^{-1}, \quad C_{eq} = \left( \frac{1}{C_1 + C_2} + \frac{1}{C_2} \right)^{-1}, \quad C_{eq} = \left( \frac{1}{C_1 + C_2 + C_3} \right)^{-1} \]

One capacitor in parallel with a series combination of the other two:

\[ C_{eq} = \left( \frac{C_1 C_2}{C_1 + C_2} \right) + C_3, \quad C_{eq} = \left( \frac{C_1 C_3}{C_1 + C_3} \right) + C_2, \quad C_{eq} = \left( \frac{C_2 C_3}{C_2 + C_3} \right) + C_1 \]

10. Nothing happens to the charge if the wires are disconnected. If the wires are connected to each other, the charge rapidly recombines, leaving the capacitor uncharged.

12. All connections of capacitors are not simple combinations of series and parallel circuits. As an example of such a complex circuit, consider the network of five capacitors \( C_1, C_2, C_3, C_4, \) and \( C_5 \) shown below.

![Network of five capacitors](image)

This combination cannot be reduced to a simple equivalent by the techniques of combining series and parallel capacitors.

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

**PROBLEM SOLUTIONS**

16.1 (a) Because the electron has a negative charge, it experiences a force in the direction opposite to the field and, when released from rest, will move in the negative \( x \) direction. The work done on the electron by the field is

\[ W = F_x (\Delta x) = (qE_x) \Delta x = \left(-1.60 \times 10^{-19} \text{ C} \right) \left( -92 \text{ N/C} \right) \left(-3.20 \times 10^{-2} \text{ m} \right) = 1.92 \times 10^{-18} \text{ J} \]

(b) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

\[ \Delta PE = -W = -1.92 \times 10^{-18} \text{ J} \]

continued on next page
Chapter 16

(c) Since the Coulomb force is a conservative force, conservation of energy gives
\[ \Delta KE + \Delta PE = 0, \text{ or } KE_f - KE_i = -\Delta PE \]
and
\[ \sqrt{-2(\Delta PE)} = \sqrt{-2\left(-1.92 \times 10^{-18} \text{ J}\right) / 9.11 \times 10^{-31} \text{ kg}} = 2.05 \times 10^6 \text{ m/s in the } -x \text{ direction} \]

16.2 (a) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,
\[ \Delta PE = -W = -\left[qE_x(\Delta x) + qE_y(\Delta y)\right] \]
\[ = -\left[q(0)\Delta x + (5.40 \times 10^{-6} \text{ C})(+327 \text{ N/C})(-32.0 \times 10^{-3} \text{ m})\right] = 5.65 \times 10^{-4} \text{ J} \]

(b) The change in the electrical potential is the change in electric potential energy per unit charge, or
\[ \Delta V \left[\frac{\Delta PE}{q}\right] = \frac{+5.65 \times 10^{-4} \text{ J}}{+5.40 \times 10^{-6} \text{ C}} = +105 \text{ V} \]

16.3 The work done by the agent moving the charge out of the cell is
\[ W_{\text{input}} = -W_{\text{field}} = -(\Delta PE_x) = +q(\Delta V) \]
\[ = \left(1.60 \times 10^{-19} \text{ C}\right)(+90 \times 10^{-3} \text{ J/C}) = 1.4 \times 10^{-20} \text{ J} \]

16.4 \[ \Delta PE_x = q(\Delta V) = q(V_f - V_i), \text{ so } q \frac{\Delta PE_x}{V_f - V_i} = \frac{-1.92 \times 10^{-17} \text{ J}}{+60.0 \text{ J/C}} = -3.20 \times 10^{-19} \text{ C} \]

16.5 \[ E = \frac{|\Delta V|}{d} = \frac{25 000 \text{ J/C}}{1.5 \times 10^{-2} \text{ m}} = 1.7 \times 10^8 \text{ N/C} \]

16.6 Since potential difference is work per unit charge \( \Delta V = \frac{W}{q} \), the work done is
\[ W = q(\Delta V) = (3.6 \times 10^5 \text{ C})(+12 \text{ J/C}) = 4.3 \times 10^6 \text{ J} \]

16.7 (a) \[ E = \frac{|\Delta V|}{d} = \frac{600 \text{ J/C}}{5.33 \times 10^{-3} \text{ m}} = 1.13 \times 10^5 \text{ N/C} \]

(b) \[ F = qE = \left(1.60 \times 10^{-19} \text{ C}\right)(1.13 \times 10^5 \text{ N/C}) = 1.80 \times 10^{-14} \text{ N} \]

(c) \[ W = F \cdot s \cos \theta \]
\[ = \left(1.80 \times 10^{-14} \text{ N}\right)[(5.33 - 2.90) \times 10^{-3} \text{ m}] \cos 0^\circ = 4.38 \times 10^{-17} \text{ J} \]
16.8 (a) Using conservation of energy, $\Delta KE + \Delta PE = 0$, with $KE_f = 0$ since the particle is “stopped,” we have

$$\Delta PE = -\Delta KE = -\left(0 - \frac{1}{2} m v_i^2\right) = +\frac{1}{2} (9.11 \times 10^{-31} \text{ kg}) (2.85 \times 10^7 \text{ m/s})^2 = +3.70 \times 10^{-16} \text{ J}$$

The required stopping potential is then

$$\Delta V = \frac{\Delta PE}{q} = \frac{+3.70 \times 10^{-16} \text{ J}}{-1.60 \times 10^{-19} \text{ C}} = -2.31 \times 10^3 \text{ V} = -2.31 \text{ kV}$$

(b) Being more massive than electrons, protons traveling at the same initial speed will have more initial kinetic energy and require a greater magnitude stopping potential.

(c) Since $\Delta V_{\text{stopping}} = \frac{\Delta PE}{q} = -\frac{-\Delta KE}{q} = -\frac{mv_i^2}{2q}$, the ratio of the stopping potential for a proton to that for an electron having the same initial speed is

$$\frac{\Delta V_p}{\Delta V_e} = \frac{-m_p v_i^2 / 2 (+e)}{-m_e v_i^2 / 2 (-e)} = \frac{m_p}{m_e}$$

16.9 (a) Use conservation of energy

$$(KE + PE_s + PE_e)_i = (KE + PE_s + PE_e)_f$$

or

$$\Delta (KE) + \Delta (PE_s) + \Delta (PE_e) = 0$$

$\Delta (KE) = 0$ since the block is at rest at both beginning and end.

$$\Delta (PE_s) = \frac{1}{2} k x_{\text{max}}^2 - 0$$

where $x_{\text{max}}$ is the maximum stretch of the spring.

$$\Delta (PE_e) = -W = -(QE) x_{\text{max}}$$

Thus, $0 + \frac{1}{2} k x_{\text{max}}^2 - (QE) x_{\text{max}} = 0$, giving

$$x_{\text{max}} = \frac{2QE}{k} = \frac{2(35.0 \times 10^{-6} \text{ C})(4.86 \times 10^4 \text{ V/m})}{78.0 \text{ N/m}} = 4.36 \times 10^{-2} \text{ m} = 4.36 \text{ cm}$$

(b) At equilibrium, $\Sigma F = F_s + F_e = 0$, or $-kx_{\text{eq}} + QE = 0$

Therefore, $x_{\text{eq}} = \frac{QE}{k} = \frac{1}{2} x_{\text{max}} = \frac{2.18 \text{ cm}}{2}$

The amplitude is the distance from the equilibrium position to each of the turning points (at $x = 0$ and $x = 4.36 \text{ cm}$), so $A = 2.18 \text{ cm} = x_{\text{max}}/2$.

continued on next page
(c) From conservation of energy, \(\Delta (KE) + \Delta (PE) + \Delta (PE_e) = 0 + \frac{1}{2} kx_{\text{max}}^2 + Q(\Delta V) = 0.\) Since \(x_{\text{max}} = 2A,\) this gives

\[
\Delta V = -\frac{kx_{\text{max}}^2}{2Q} = -\frac{k(2A)^2}{2Q} \quad \text{or} \quad \Delta V = -\frac{2kA^2}{Q}
\]

16.10 Using \(\Delta y = v_{y0}t + \frac{1}{2} a_y t^2\) for the full flight gives

\[
0 = v_{y0}t + \frac{1}{2} a_y t^2, \quad \text{or} \quad a_y = \frac{-2v_{y0}}{t}
\]

Then, using \(v_f^2 = v_i^2 + 2a_y(\Delta y)\) for the upward part of the flight gives

\[
(\Delta y)_{\text{max}} = 0 - \frac{v_{y0}^2}{2a_y} = \frac{v_{y0} t}{2(2v_{y0}/t)} = \frac{(20.1 \text{ m/s})(4.10 \text{ s})}{4} = 20.6 \text{ m}
\]

From Newton’s second law, \(a_y = \frac{\Sigma F_y}{m} = \frac{-mg - qE}{m} = -\left(g + \frac{qE}{m}\right).\) Equating this to the earlier result gives \(a_y = -\left(g + \frac{qE}{m}\right) = -\frac{2v_{y0}}{t},\) so the electric field strength is

\[
E = \left(\frac{m}{q}\right)\left[\frac{2v_{y0}}{t} - g\right] = \left(\frac{2.00 \text{ kg}}{5.00 \times 10^{-7} \text{ C}}\right)\left[\frac{2(20.1 \text{ m/s})}{4.10 \text{ s}} - 9.80 \text{ m/s}^2\right] = 1.95 \times 10^3 \text{ N/C}
\]

Thus, \((\Delta V)_{\text{max}} = (\Delta y_{\text{max}})E = (20.6 \text{ m})(1.95 \times 10^3 \text{ N/C}) = 4.02 \times 10^4 \text{ V} = \boxed{40.2 \text{ kV}}\)

16.11

(a) \(V_a = \frac{k\alpha}{r_a} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^{-2} \cdot \text{C}^{-2})(-1.60 \times 10^{-19} \text{ C})}{0.250 \times 10^{-2} \text{ m}} = -5.75 \times 10^{-7} \text{ V}\)

(b) \(V_b = \frac{k\alpha}{r_b} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^{-2} \cdot \text{C}^{-2})(-1.60 \times 10^{-19} \text{ C})}{0.750 \times 10^{-2} \text{ m}} = -1.92 \times 10^{-7} \text{ V}\)

\(\Delta V = V_b - V_a = -1.92 \times 10^{-7} \text{ V} - (-5.75 \times 10^{-7} \text{ V}) = \boxed{3.83 \times 10^{-7} \text{ V}}\)

(c) The original electron will be repelled by the negatively charged particle which suddenly appears at point \(A.\) Unless the electron is fixed in place, it will move in the opposite direction, away from points \(A\) and \(B,\) thereby lowering the potential difference between these points.

16.12

(a) At the origin, the total potential is

\[
V_{\text{origin}} = \frac{k\alpha}{r_i} + \frac{k\beta}{r_f}
\]

\[= (8.99 \times 10^9 \text{ N} \cdot \text{m}^{-2} \cdot \text{C}^{-2}) \left[\frac{4.50 \times 10^{-6} \text{ C}}{1.25 \times 10^{-2} \text{ m}} + \frac{(-2.24 \times 10^{-6} \text{ C})}{1.80 \times 10^{-2} \text{ m}}\right] = \boxed{2.12 \times 10^9 \text{ V}}
\]

continued on next page
(b) At point \( B \) located at \((1.50 \text{ cm}, 0)\), the needed distances are

\[
r_1 = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2} = \sqrt{(1.50 \text{ cm})^2 + (1.25 \text{ cm})^2} = 1.95 \text{ cm}
\]

and

\[
r_2 = \sqrt{(x_0 - x_2)^2 + (y_0 - y_2)^2} = \sqrt{(1.50 \text{ cm})^2 + (1.80 \text{ cm})^2} = 2.34 \text{ cm}
\]

giving

\[
V_k = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m/C}^2\right) \left(\frac{4.50 \times 10^{-6} \text{ C}}{1.95 \times 10^{-2} \text{ m}} + \frac{-2.24 \times 10^{-6} \text{ C}}{2.34 \times 10^{-2} \text{ m}}\right) = 1.21 \times 10^5 \text{ V}
\]

16.13 (a) Calling the 2.00 \( \mu \text{C} \) charge \( q_s \),

\[
V = \sum \frac{kq}{r} = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}} \right)
\]

\[
= \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left(\frac{8.00 \times 10^{-6} \text{ C}}{0.060 \text{ m}} + \frac{4.00 \times 10^{-6} \text{ C}}{0.030 \text{ m}} + \frac{2.00 \times 10^{-6} \text{ C}}{\sqrt{(0.060 \text{ m})^2 + (0.030 \text{ m})^2}}\right)
\]

\[
V = 2.67 \times 10^6 \text{ V}
\]

(b) Replacing \( 2.00 \times 10^{-6} \text{ C} \) by \( -2.00 \times 10^{-6} \text{ C} \) in part (a) yields

\[
V = 2.13 \times 10^6 \text{ V}
\]

16.14 \( W = q(\Delta V) = q(V_f - V_i) \), and

\[
V_i = 0 \text{ since the 8.00 } \mu\text{C is infinite distance from other charges.}
\]

\[
V_f = k \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right) = \left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \left(\frac{2.00 \times 10^{-6} \text{ C}}{0.030 \text{ m}} + \frac{4.00 \times 10^{-6} \text{ C}}{\sqrt{(0.030 \text{ m})^2 + (0.060 \text{ m})^2}}\right)
\]

\[
= 1.135 \times 10^6 \text{ V}
\]

Thus, \( W = (8.00 \times 10^{-6} \text{ C})(0 - 1.135 \times 10^6 \text{ V}) = -9.08 \text{ J} \)
(a) \[ V = \sum_{i} \frac{k_{q_{i}}}{r_{i}} \]
\[ = \left( 8.99 \times 10^{9} \ \text{N} \cdot \text{m}^{2}/\text{C}^{2} \right) \left( \frac{5.00 \times 10^{-9} \ \text{C}}{0.175 \ \text{m}} - \frac{3.00 \times 10^{-9} \ \text{C}}{0.175 \ \text{m}} \right) = 103 \ \text{V} \]

(b) \[ PE = \frac{k_{q_{1}}q_{2}}{r_{12}} \]
\[ = \left( 8.99 \times 10^{9} \ \text{N} \cdot \text{m}^{2}/\text{C}^{2} \right) \left( 5.00 \times 10^{-9} \ \text{C} \right) \left( -3.00 \times 10^{-9} \ \text{C} \right) = -3.85 \times 10^{-7} \ \text{J} \]

The negative sign means that positive work must be done to separate the charges (that is, bring them up to a state of zero potential energy).

16.16 The potential at distance \( r = 0.300 \ \text{m} \) from a charge \( Q = +9.00 \times 10^{-9} \ \text{C} \) is
\[ V = \frac{k_{Q}}{r} = \frac{\left( 8.99 \times 10^{9} \ \text{N} \cdot \text{m}^{2}/\text{C}^{2} \right) \left( 9.00 \times 10^{-9} \ \text{C} \right)}{0.300 \ \text{m}} = +270 \ \text{V} \]

Thus, the work required to carry a charge \( q = 3.00 \times 10^{-9} \ \text{C} \) from infinity to this location is
\[ W = qV = \left( 3.00 \times 10^{-9} \ \text{C} \right) \left( +270 \ \text{V} \right) = 8.09 \times 10^{-7} \ \text{J} \]

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as
\[ r_{5} = \sqrt{(4.00 \ \text{cm})^{2} - (1.00 \ \text{cm})^{2}} = \sqrt{15} \ \text{cm} = \sqrt{15} \times 10^{-2} \ \text{m} \]

Then, the potential at the midpoint of the base is \( V = \sum_{i} \frac{k_{q_{i}}}{r_{i}} \), or
\[ V = \left( 8.99 \times 10^{9} \ \text{N} \cdot \text{m}^{2}/\text{C}^{2} \right) \left( \frac{-7.00 \times 10^{-9} \ \text{C}}{0.010 \ \text{m}} + \frac{-7.00 \times 10^{-9} \ \text{C}}{0.010 \ \text{m}} + \frac{+7.00 \times 10^{-9} \ \text{C}}{\sqrt{15} \times 10^{-2} \ \text{m}} \right) \]
\[ = -1.10 \times 10^{4} \ \text{V} = -11.0 \ \text{kV} \]
Outside the spherical charge distribution, the potential is the same as for a point charge at the center of the sphere,

\[ V = kQ/r, \text{ where } Q = 1.00 \times 10^{-9} \text{ C} \]

Thus, \( \Delta(PE_e) = q(\Delta V) = -ekQ \left( \frac{1}{r_f} - \frac{1}{r_i} \right) \)

and from conservation of energy, \( \Delta(KE) = -\Delta(PE_e) \),

or \( \frac{1}{2}mv^2 = - \left[ -ekQ \left( \frac{1}{r_f} - \frac{1}{r_i} \right) \right]\). This gives \( v = \sqrt{\frac{2kQe \left( \frac{1}{r_f} - \frac{1}{r_i} \right)}{m}} \), or

\[ v = \frac{7.25 \times 10^{6}}{s} \text{ m/s} \]

When the charge configuration consists of only the two protons \( q_1 \) and \( q_2 \) in the sketch, the potential energy of the configuration is

\[ PE_a = \frac{kq_1q_2}{r_{12}} = \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left(1.00 \times 10^{-9} \text{ C}\right)^2}{6.00 \times 10^{-15} \text{ m}} \]

or \( PE_a = 3.84 \times 10^{-14} \text{ J} \)

When the alpha particle \( q_3 \) in the sketch is added to the configuration, there are three distinct pairs of particles, each of which possesses potential energy. The total potential energy of the configuration is now

\[ PE_a = \frac{kq_1q_2}{r_{12}} + \frac{kq_3q_1}{r_{31}} + \frac{kq_3q_2}{r_{32}} = PE_a + 2 \left( \frac{k \left(2e^2\right)}{r_{31}} \right) \]

where use has been made of the facts that \( q_1q_3 = q_2q_3 = e(2e) = 2e^2 \) and

\[ r_{13} = r_{32} = \sqrt{(3.00 \text{ fm})^2 + (3.00 \text{ fm})^2} = 4.24 \text{ fm} = 4.24 \times 10^{-15} \text{ m} \]

Also, note that the first term in this computation is just the potential energy computed in part (a). Thus,

\[ PE_a = PE_a + 4 \frac{k \left(2e^2\right)}{r_{31}} \]

\[ = 3.84 \times 10^{-14} \text{ J} + 4 \frac{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \left(1.60 \times 10^{-19} \text{ C}\right)^2}{4.24 \times 10^{-15} \text{ m}} = 2.55 \times 10^{-13} \text{ J} \]

continued on next page
(c) If we start with the three-particle system of part (b) and allow the alpha particle to escape to infinity [thereby returning us to the two-particle system of part (a)], the change in electric potential energy will be
\[
\Delta PE = PE_a - PE_e = 3.84 \times 10^{-14} \, \text{J} - 2.55 \times 10^{-13} \, \text{J} = -2.17 \times 10^{-13} \, \text{J}
\]

(d) Conservation of energy, \( \Delta KE + \Delta PE = 0 \), gives the speed of the alpha particle at infinity in the situation of part (c) as
\[
v_a = \sqrt{\frac{-2(\Delta PE)}{m_a}} = \sqrt{\frac{-2(-2.17 \times 10^{-13} \, \text{J})}{6.64 \times 10^{-27} \, \text{kg}}} = 8.08 \times 10^6 \, \text{m/s}
\]

(e) When, starting with the three-particle system, the two protons are both allowed to escape to infinity, there will be no remaining pairs of particles and hence no remaining potential energy. Thus, \( \Delta PE = 0 - PE_i = PE_a \), and conservation of energy gives the change in kinetic energy as \( \Delta KE = -\Delta PE = +PE_i \). Since the protons are identical particles, this increase in kinetic energy is split equally between them giving
\[
KE_{proton} = \frac{1}{2} m_p v_p^2 = \frac{1}{2} (PE_i)
\]
or
\[
v_p = \sqrt{\frac{PE_i}{m_p}} = \sqrt{\frac{2.55 \times 10^{-13} \, \text{J}}{1.67 \times 10^{-27} \, \text{kg}}} = 1.24 \times 10^7 \, \text{m/s}
\]

16.20 (a) If a proton and an alpha particle, initially at rest 4.00 fm apart, are released and allowed to recede to infinity, the final speeds of the two particles will differ because of the difference in the masses of the particles. Thus, attempting to solve for the final speeds by use of conservation of energy alone leads to a situation of having [one equation with two unknowns], and does not permit a solution.

(b) In the situation described in part (a) above, one can obtain a second equation with the two unknown final speeds by using [conservation of linear momentum]. Then, one would have two equations which could be solved simultaneously both unknowns.

*continued on next page*
(c) From conservation of energy:
\[
\left( \frac{1}{2} m_a v_a^2 + \frac{1}{2} m_p v_p^2 \right) - \left( 0 - \frac{kq_a q_p}{r} \right) = 0
\]

or
\[
m_a v_a^2 + m_p v_p^2 = \frac{2kq_a q_p}{r}
\]

yielding
\[
m_a v_a^2 + m_p v_p^2 = 2.30 \times 10^{-11} \text{ J}
\]  

From conservation of linear momentum,
\[
m_a v_a + m_p v_p = 0 \quad \text{or} \quad |v_a| = \left( \frac{m_p}{m_a} \right) v_p
\]

\[
m_a \left( \frac{m_p}{m_a} \right) v_p^2 + m_p v_p^2 = 2.30 \times 10^{-11} \text{ J} \quad \text{or} \quad \left( \frac{m_p}{m_a} + 1 \right) m_p v_p^2 = 2.30 \times 10^{-11} \text{ J}
\]

and
\[
v_p = \sqrt{\frac{2.30 \times 10^{-11} \text{ J}}{(m_p/m_a + 1)m_p}} = \sqrt{\frac{2.30 \times 10^{-11} \text{ J}}{(1.67 \times 10^{-27}/6.64 \times 10^{-27} + 1)(1.67 \times 10^{-27} \text{ kg})}} = 1.05 \times 10^7 \text{ m/s}
\]

Then, Equation [2] gives the final speed of the alpha particle as
\[
|v_a| = \left( \frac{m_p}{m_a} \right) v_p = \left( \frac{1.67 \times 10^{-27} \text{ kg}}{6.64 \times 10^{-27} \text{ kg}} \right)(1.05 \times 10^7 \text{ m/s}) = 2.64 \times 10^6 \text{ m/s}
\]

16.21 \( V = \frac{kQ}{r} \) so
\[
r = \frac{kQ}{V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(8.00 \times 10^{-9} \text{ C})}{71.9 \text{ V} \cdot \text{m}} = 71.9 \text{ V} \cdot \text{m} / V
\]

For \( V = 100 \text{ V}, 50.0 \text{ V}, \) and \( 25.0 \text{ V} \), \( r = 0.719 \text{ m}, 1.44 \text{ m}, \) and \( 2.88 \text{ m} \)

The radii are \textbf{inversely proportional} to the potential.

16.22 By definition, the work required to move a charge from one point to any other point on an equipotential surface is zero. From the definition of work, \( W = (F \cos \theta) \cdot s \), the work is zero only if \( s = 0 \) or \( F \cos \theta = 0 \). The displacement \( s \) cannot be assumed to be zero in all cases. Thus, one must require that \( F \cos \theta = 0 \). The force \( F \) is given by \( F = qE \) and neither the charge \( q \) nor the field strength \( E \) can be assumed to be zero in all cases. Therefore, the only way the work can be zero in all cases is if \( \cos \theta = 0 \). But if \( \cos \theta = 0 \), then \( \theta = 90^\circ \) or the force (and hence the electric field) must be perpendicular to the displacement \( s \) (which is tangent to the surface). That is, the field must be perpendicular to the equipotential surface at all points on that surface.
16.23 From conservation of energy, \((KE + PE_e)_f = (KE + PE_e)_i\), which gives

\[
0 + \frac{kQq}{r_f} = \frac{1}{2} m_e v_i^2 + 0 \quad \text{or} \quad r_f = \frac{2kQq}{m_e v_i^2} = \frac{2k_e (79e)(2e)}{m_e v_i^2}
\]

\[
r_f = \frac{2 \left( 8.99 \times 10^9 \ \text{N m}^2 \text{C}^2 \right) \left( 158 \right) \left( 1.60 \times 10^{-19} \ \text{C} \right)^2}{\left( 6.64 \times 10^{-27} \ \text{kg} \right) \left( 2.00 \times 10^7 \ \text{m/s} \right)^2} = 2.74 \times 10^{-14} \ \text{m}
\]

16.24 (a) The distance from any one of the corners of the square to the point at the center is one half the length of the diagonal of the square, or

\[
r = \frac{\text{diagonal}}{2} = \frac{\sqrt{a^2 + a^2}}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}
\]

Since the charges have equal magnitudes and are all the same distance from the center of the square, they make equal contributions to the total potential. Thus,

\[
V_{\text{total}} = 4V_{\text{single charge}} = 4 \frac{kQ}{r} = 4 \frac{kQ}{a/\sqrt{2}} = 4\sqrt{2}k_e Q/a
\]

(b) The work required to carry charge \(q\) from infinity to the point at the center of the square is equal to the increase in the electric potential energy of the charge, or

\[
W = PE_{\text{center}} - PE_e = qV_{\text{total}} - 0 = q \left( 4\sqrt{2}k_e Q/a \right) = 4\sqrt{2}k_e qQ/a
\]

16.25 (a) \(C = \varepsilon_0 \frac{A}{d} = \frac{8.85 \times 10^{-12} \ \text{C}^2 \text{N}^{-1} \text{m}^{-2}}{(1.0 \times 10^6 \ \text{m}^2)} = 1.1 \times 10^{-8} \ \text{F}\)

(b) \(Q_{\text{max}} = C(\Delta V)_{\text{max}} = C(E_{\text{max}}d) \]

\[
= (1.11 \times 10^{-8} \ \text{F})(3.0 \times 10^6 \ \text{N/C})(800 \ \text{m}) = 27 \ \text{C}
\]

16.26 (a) \(C = \frac{Q}{\Delta V} = \frac{27.0 \ \mu\text{C}}{9.00 \ \text{V}} = 3.00 \ \mu\text{F}\)

(b) \(Q = C(\Delta V) = (3.00 \ \mu\text{F})(12.0 \ \text{V}) = 36.0 \ \mu\text{C}\)

16.27 (a) The capacitance of this air filled (dielectric constant, \(\kappa = 1.00\)) parallel plate capacitor is

\[
C = \frac{k \varepsilon_0 A}{d} = \frac{(1.00)(8.85 \times 10^{-12} \ \text{C}^2 / \text{N} \cdot \text{m}^2)(2.30 \times 10^{-4} \ \text{m}^3)}{1.50 \times 10^{-3} \ \text{m}} = 1.36 \times 10^{-13} \ \text{F} = 1.36 \ \text{pF}
\]

continued on next page
(b) \( Q = C(\Delta V) = (1.36 \times 10^{-12} \text{ F})(12.0 \text{ V}) = 1.63 \times 10^{-11} \text{ C} = 16.3 \text{ pC} \)

(c) \( E = \frac{\Delta V}{d} = \frac{12.0 \text{ V}}{1.50 \times 10^{-3} \text{ m}} = 8.00 \times 10^{3} \text{ V/m} = 8.00 \times 10^{3} \text{ N/C} \)

16.28
(a) \( Q = C(\Delta V) = (4.00 \times 10^{-6} \text{ F})(12.0 \text{ V}) = 48.0 \times 10^{-6} \text{ C} = 48.0 \mu\text{C} \)

(b) \( Q = C(\Delta V) = (4.00 \times 10^{-6} \text{ F})(1.50 \text{ V}) = 6.00 \times 10^{-6} \text{ C} = 6.00 \mu\text{C} \)

16.29
(a) \( E = \frac{\Delta V}{d} = \frac{20.0 \text{ V}}{1.80 \times 10^{-3} \text{ m}} = 1.11 \times 10^{4} \text{ V/m} = 11.1 \text{ kV/m} \) directed toward the negative plate

(b) \( C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.60 \times 10^{-4} \text{ m}^2)}{1.80 \times 10^{-3} \text{ m}} = 3.74 \times 10^{-12} \text{ F} = 3.74 \text{ pF} \)

(c) \( Q = C(\Delta V) = (3.74 \times 10^{-12} \text{ F})(20.0 \text{ V}) = 7.47 \times 10^{-11} \text{ C} = 74.7 \text{ pC} \) on one plate and \(-74.7 \text{ pC} \) on the other plate.

16.30
\[ C = \frac{\varepsilon_0 A}{d}, \text{ so} \]
\[ d = \frac{\varepsilon_0 A}{C} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(21.0 \times 10^{-12} \text{ m}^2)}{60.0 \times 10^{-15} \text{ F}} = 3.10 \times 10^{-9} \text{ m} \]
\[ d = (3.10 \times 10^{-9} \text{ m})\left(\frac{1 \text{ Å}}{10^{-10} \text{ m}}\right) = 31.0 \text{ Å} \]

16.31
(a) Assuming the capacitor is air-filled (\( \kappa = 1 \)), the capacitance is
\[ C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.200 \text{ m}^2)}{3.00 \times 10^{-3} \text{ m}} = 5.90 \times 10^{-10} \text{ F} \]

(b) \( Q = C(\Delta V) = (5.90 \times 10^{-10} \text{ F})(6.00 \text{ V}) = 3.54 \times 10^{-9} \text{ C} \)

(c) \( E = \frac{\Delta V}{d} = \frac{6.00 \text{ V}}{3.00 \times 10^{-3} \text{ m}} = 2.00 \times 10^{3} \text{ V/m} = 2.00 \times 10^{3} \text{ N/C} \)

(d) \( \sigma = \frac{Q}{A} = \frac{3.54 \times 10^{-9} \text{ C}}{0.200 \text{ m}^2} = 1.77 \times 10^{-4} \text{ C/m}^2 \)

(e) Increasing the distance separating the plates decreases the capacitance, the charge stored, and the electric field strength between the plates. This means that all of the previous answers will be decreased.
16.32  \[ \Sigma F_y = 0 \Rightarrow T \cos 15.0^\circ = mg \quad \text{or} \quad T = \frac{mg}{\cos 15.0^\circ} \]
\[ \Sigma F_x = 0 \Rightarrow qE = T \sin 15.0^\circ = mg \tan 15.0^\circ \]

or  \[ E = \frac{mg \tan 15.0^\circ}{q} \]
\[ \Delta V = Ed = \frac{mgd \tan 15.0^\circ}{q} \]
\[ \Delta V = \frac{(350 \times 10^{-6} \text{ kg})(9.80 \text{ m/s}^2)(0.040 \text{ m}) \tan 15.0^\circ}{30.0 \times 10^{-3} \text{ C}} = 1.23 \times 10^3 \text{ V} = 1.23 \text{ kV} \]

16.33  (a)  Capacitors in a series combination store the same charge, \( Q = C_{eq} (\Delta V) \), where \( C_{eq} \) is the equivalent capacitance and \( \Delta V \) is the potential difference maintained across the series combination. The equivalent capacitance for the given series combination is

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}, \quad \text{or} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}, \quad \text{giving} \]
\[ C_{eq} = \frac{(2.50 \mu F)(6.25 \mu F)}{2.50 \mu F + 6.25 \mu F} = 1.79 \mu F \]

so the charge stored on each capacitor in the series combination is

\[ Q = C_{eq} (\Delta V) = (1.79 \mu F)(6.00 \text{ V}) = 10.7 \mu C \]

(b)  When connected in parallel, each capacitor has the same potential difference, \( \Delta V = 6.00 \text{ V} \), maintained across it. The charge stored on each capacitor is then

For \( C_1 = 2.50 \mu F \):  \[ Q_1 = C_1 (\Delta V) = (2.50 \mu F)(6.00 \text{ V}) = 15.0 \mu C \]

For \( C_2 = 6.25 \mu F \):  \[ Q_2 = C_2 (\Delta V) = (6.25 \mu F)(6.00 \text{ V}) = 37.5 \mu C \]

16.34  (a)  When connected in series, the equivalent capacitance is \[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}, \quad \text{or} \]
\[ C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.20 \mu F)(8.50 \mu F)}{4.20 \mu F + 8.50 \mu F} = 2.81 \mu F \]

(b)  When connected in parallel, the equivalent capacitance is

\[ C_{eq} = C_1 + C_2 = 4.20 \mu F + 8.50 \mu F = 12.7 \mu F \]
16.35 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance, \( C_{bc} = 2.00 \, \mu F + 6.00 \, \mu F = 8.00 \, \mu F \). Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore

\[
\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00 \, \mu F}
\]

giving

\[
C_{eq} = \frac{8.00 \, \mu F}{3} = 2.67 \, \mu F
\]

(b) The charge stored on each capacitor in the series combination is

\[
Q_{ab} = Q_{bc} = Q_{cd} = C_{eq} (\Delta V_{ab}) = (2.67 \, \mu F)(9.00 \, V) = 24.0 \, \mu C
\]

Then, note that \( \Delta V_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0 \, \mu C}{8.00 \, \mu F} = 3.00 \, V \). The charge on each capacitor in the original circuit is:

On the 8.00 \, \mu F between a and b: \( Q_a = Q_{ab} = [24.0 \, \mu C] \)

On the 8.00 \, \mu F between c and d: \( Q_b = Q_{cd} = [24.0 \, \mu C] \)

On the 2.00 \, \mu F between b and c: \( Q_c = C_c (\Delta V_{bc}) = (2.00 \, \mu F)(3.00 \, V) = [6.00 \, \mu C] \)

On the 6.00 \, \mu F between b and c: \( Q_d = C_d (\Delta V_{bc}) = (6.00 \, \mu F)(3.00 \, V) = [18.0 \, \mu C] \)

(c) Note that \( \Delta V_a = \Delta V_2 = \Delta V_6 = \Delta V_3 = 3.00 \, V \), and that \( \Delta V_3 = \frac{Q_3}{C_3} = 24.0 \, \mu C/8.00 \, \mu F = 3.00 \, V \). We earlier found that \( \Delta V_{bc} = 3.00 \, V \), so we conclude that the potential difference across each capacitor in the circuit is

\[
\Delta V_a = \Delta V_2 = \Delta V_6 = \Delta V_3 = 3.00 \, V
\]

16.36 \( C_{parallel} = C_1 + C_2 = 9.00 \, pF \Rightarrow C_1 = 9.00 \, pF - C_2 \) [1]

\[
\frac{1}{C_{parallel}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{parallel} = \frac{C_1 C_2}{C_1 + C_2} = 2.00 \, pF
\]

Thus, using Equation [1],

\[
C_{parallel} = \frac{(9.00 \, pF - C_2) C_2}{(9.00 \, pF - C_2) + C_2} = 2.00 \, pF, \text{ which reduces to}
\]

\[
C_2^2 - (9.00 \, pF)C_2 + 18.0 \, (pF)^2 = 0, \text{ or } (C_2 - 6.00 \, pF)(C_2 - 3.00 \, pF) = 0
\]

Therefore, either \( C_2 = 6.00 \, pF \) and, from Equation [1], \( C_1 = 3.00 \, pF \)

or \( C_2 = 3.00 \, pF \) and \( C_1 = 6.00 \, pF \).

We conclude that the two capacitances are \( 3.00 \, pF \) and \( 6.00 \, pF \).
16.37 (a) The equivalent capacitance of the series combination in the upper branch is
\[
\frac{1}{C_{\text{upper}}} = \frac{1}{3.00 \ \mu F} + \frac{1}{6.00 \ \mu F} = \frac{2 + 1}{6.00 \ \mu F}
\]
or
\[C_{\text{upper}} = 2.00 \ \mu F\]
Likewise, the equivalent capacitance of the series combination in the lower branch is
\[
\frac{1}{C_{\text{lower}}} = \frac{1}{2.00 \ \mu F} + \frac{1}{4.00 \ \mu F} = \frac{2 + 1}{4.00 \ \mu F} \quad \text{or} \quad C_{\text{lower}} = 1.33 \ \mu F
\]
These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is
\[
C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00 \ \mu F + 1.33 \ \mu F = 3.33 \ \mu F
\]
(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. The charge stored on each capacitor in the series combination in the upper branch is
\[
Q_1 = Q_6 = Q_{\text{upper}} \left( \Delta V \right) = (2.00 \ \mu F)(90.0 \ \text{V}) = 180 \ \mu C
\]
and the charge stored on each capacitor in the series combination in the lower branch is
\[
Q_3 = Q_6 = Q_{\text{lower}} \left( \Delta V \right) = (1.33 \ \mu F)(90.0 \ \text{V}) = 120 \ \mu C
\]
(c) The potential difference across each of the capacitors in the circuit is:
\[
\Delta V_1 = \frac{Q_1}{C_1} = \frac{120 \ \mu C}{2.00 \ \mu F} = 60.0 \ \text{V} \quad \Delta V_2 = \frac{Q_2}{C_2} = \frac{120 \ \mu C}{4.00 \ \mu F} = 30.0 \ \text{V}
\]
\[
\Delta V_3 = \frac{Q_3}{C_3} = \frac{180 \ \mu C}{3.00 \ \mu F} = 60.0 \ \text{V} \quad \Delta V_4 = \frac{Q_4}{C_4} = \frac{180 \ \mu C}{6.00 \ \mu F} = 30.0 \ \text{V}
\]
16.38 (a) The equivalent capacitance of the series combination in the rightmost branch of the circuit is

$$\frac{1}{C_{\text{right}}} = \frac{1}{24.0 \ \mu F} + \frac{1}{8.00 \ \mu F} = \frac{1 + 3}{24.0 \ \mu F}$$

or

$$C_{\text{right}} = 6.00 \ \mu F$$

(b) The equivalent capacitance of the three capacitors now connected in parallel with each other and with the battery is

$$C_{\text{eq}} = 4.00 \ \mu F + 2.00 \ \mu F + 6.00 \ \mu F = 12.0 \ \mu F$$

(c) The total charge stored in this circuit is

$$Q_{\text{total}} = C_{\text{eq}} (\Delta V) = (12.0 \ \mu F)(36.0 \ V)$$

or

$$Q_{\text{total}} = 432 \ \mu C$$

(d) The charges on the three capacitors shown in Diagram 1 are:

$$Q_1 = C_1 (\Delta V) = (4.00 \ \mu F)(36.0 \ V) = 144 \ \mu C$$

$$Q_2 = C_2 (\Delta V) = (2.00 \ \mu F)(36.0 \ V) = 72 \ \mu C$$

$$Q_{\text{right}} = C_{\text{right}} (\Delta V) = (6.00 \ \mu F)(36.0 \ V) = 216 \ \mu C$$

Yes, $$Q_4 + Q_2 + Q_{\text{right}} = Q_{\text{total}}$$ as it should.

(e) The charge on each capacitor in the series combination in the rightmost branch of the original circuit (Figure P16.38) is

$$Q_{24} = Q_3 = Q_{\text{right}} = 216 \ \mu C$$

(f) $$\Delta V_{24} = \frac{Q_{24}}{C_{24}} = \frac{216 \ \mu C}{24.0 \ \mu F} = 9.00 \ V$$

(g) $$\Delta V_k = \frac{Q_k}{C_k} = \frac{216 \ \mu C}{8.00 \ \mu F} = 27.0 \ V$$

Note that $$\Delta V_k + \Delta V_{24} = \Delta V = 36.0 \ V$$ as it should.
The circuit may be reduced in steps as shown above.

Using Figure 3, \( Q_{ac} = \left( 4.00 \, \mu F \right) (24.0 \, \text{V}) = 96.0 \, \mu \text{C} \)

Then, in Figure 2, \( (\Delta V)_{ab} = \frac{Q_{ac}}{C_{ab}} = \frac{96.0 \, \mu \text{C}}{6.00 \, \mu \text{F}} = 16.0 \, \text{V} \)

and \( (\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0 \, \text{V} - 16.0 \, \text{V} = 8.00 \, \text{V} \)

Finally, using Figure 1, \( Q_i = C_i (\Delta V)_{ab} = \left( 1.00 \, \mu F \right) (16.0 \, \text{V}) = 16.0 \, \mu \text{C} \)

\[ Q_i = (5.00 \, \mu F) (\Delta V)_{ab} = 80.0 \, \mu \text{C}, \quad Q_i = (8.00 \, \mu F) (\Delta V)_{ac} = 64.0 \, \mu \text{C} \]

and \( Q_i = (4.00 \, \mu F) (\Delta V)_{bc} = 32.0 \, \mu \text{C} \)

16.40 From \( Q = C (\Delta V) \), the initial charge of each capacitor is

\( Q_{10} = \left( 10.0 \, \mu F \right) (12.0 \, \text{V}) = 120 \, \mu \text{C} \) and \( Q_{1} = C_{1} (0) = 0 \)

After the capacitors are connected in parallel, the potential difference across each is \( \Delta V' = 3.00 \, \text{V} \), and the total charge of \( Q = Q_{10} + Q_{1} = 120 \, \mu \text{C} \) is divided between the two capacitors as

\( Q_{10}' = \left( 10.0 \, \mu F \right) (3.00 \, \text{V}) = 30.0 \, \mu \text{C} \) and

\( Q_{1}' = Q - Q_{10}' = 120 \, \mu \text{C} - 30.0 \, \mu \text{C} = 90.0 \, \mu \text{C} \)

Thus, \( C_1 = \frac{Q_{1}'}{\Delta V'} = \frac{90.0 \, \mu \text{C}}{3.00 \, \text{V}} = 30.0 \, \mu \text{F} \)
16.41  (a) From \( Q = C(\Delta V) \), \( Q_{25} = (25.0 \ \mu F)(50.0 \ \text{V}) = 1.25 \times 10^3 \ \mu C = 1.25 \ \text{mC} \)
and \( Q_{40} = (40.0 \ \mu F)(50.0 \ \text{V}) = 2.00 \times 10^3 \ \mu C = 2.00 \ \text{mC} \)

(b) When the two capacitors are connected in parallel, the equivalent capacitance is

\[ C_{eq} = C_1 + C_2 = 25.0 \ \mu F + 40.0 \ \mu F = 65.0 \ \mu F. \]

Since the negative plate of one was connected to the positive plate of the other, the total charge stored in the parallel combination is

\[ Q = Q_{40} - Q_{25} = 2.00 \times 10^3 \ \mu C - 1.25 \times 10^3 \ \mu C = 750 \ \mu C \]

The potential difference across each capacitor of the parallel combination is

\[ \Delta V = \frac{Q}{C_{eq}} = \frac{750 \ \mu C}{65.0 \ \mu F} = 11.5 \ \text{V} \]

and the final charge stored in each capacitor is

\[ Q'_1 = C_1(\Delta V) = (25.0 \ \mu F)(11.5 \ \text{V}) = 288 \ \mu C \]

and \( Q'_0 = Q - Q'_1 = 750 \ \mu C - 288 \ \mu C = 462 \ \mu C \)

16.42  (a) The original circuit reduces to a single equivalent capacitor in the steps shown below.

\[ C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00 \ \mu F} + \frac{1}{10.0 \ \mu F}\right)^{-1} = 3.33 \ \mu F \]

\[ C_{p1} = C_1 + C_3 + C_s = 2 \times 3.33 \ \mu F + 2.00 \ \mu F = 8.66 \ \mu F \]

\[ C_{p2} = C_2 + C_s = 2 \times 10.0 \ \mu F = 20.0 \ \mu F \]

\[ C_{eq} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66 \ \mu F} + \frac{1}{20.0 \ \mu F}\right)^{-1} = 6.04 \ \mu F \]

continued on next page
(b) The total charge stored between points \(a\) and \(b\) is

\[
Q_{\text{total}} = C_\text{eq} (\Delta V)_\text{ab} = (6.04 \ \mu F)(60.0 \ \text{V}) = 362 \ \mu \text{C}
\]

Then, looking at the third figure, observe that the charges of the series capacitors of that figure are \(Q_{p1} = Q_{p2} = Q_{\text{total}} = 362 \ \mu \text{C}\). Thus, the potential difference across the upper parallel combination shown in the second figure is

\[
(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362 \ \mu \text{C}}{8.66 \ \mu \text{F}} = 41.8 \ \text{V}
\]

Finally, the charge on \(C_3\) is

\[
Q_3 = C_3 (\Delta V)_{p1} = (2.00 \ \mu \text{F})(41.8 \ \text{V}) = 83.6 \ \mu \text{C}
\]

16.43 From \(Q = C(\Delta V)\), the initial charge of each capacitor is

\[
Q_1 = (1.00 \ \mu \text{F})(10.0 \ \text{V}) = 10.0 \ \mu \text{C} \quad \text{and} \quad Q_2 = (2.00 \ \mu \text{F})(0) = 0
\]

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

\[
\Delta V = \frac{Q_1'}{1.00 \ \mu \text{F}} = \frac{Q_2'}{2.00 \ \mu \text{F}} \quad \text{or} \quad Q_1' = 2 Q_2'
\]

From conservation of charge, \(Q_1' + Q_2' = Q_1 + Q_2 = 10.0 \ \mu \text{C}\). Then, substituting from Equation [1], this becomes

\[
Q_1' + 2 Q_2' = 10.0 \ \mu \text{C}, \quad \text{giving} \quad Q_2' = \frac{10}{3} \ \mu \text{C}
\]

Finally, from Equation [1], \(Q_1' = \frac{20}{3} \ \mu \text{C}\)

16.44 Recognize that the 7.00 \(\mu\)F and the 5.00 \(\mu\)F of the center branch are connected in series. The total capacitance of that branch is

\[
C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92 \ \mu \text{F}
\]

Then recognize that this capacitor, the 4.00 \(\mu\)F capacitor, and the 6.00 \(\mu\)F capacitor are all connected in parallel between points \(a\) and \(b\). Thus, the equivalent capacitance between points \(a\) and \(b\) is

\[
C_{\text{eq}} = 4.00 \ \mu \text{F} + 2.92 \ \mu \text{F} + 6.00 \ \mu \text{F} = 12.9 \ \mu \text{F}
\]
16.45 Energy stored = \( \frac{Q^2}{2C} = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} (4.50 \times 10^{-6} \text{ F}) (12.0 \text{ V})^2 = 3.24 \times 10^{-4} \text{ J} \)

16.46 (a) The equivalent capacitance of a series combination of \( C_1 \) and \( C_2 \) is

\[ \frac{1}{C_{\text{eq}}} = \frac{1}{18.0 \mu \text{F}} + \frac{1}{36.0 \mu \text{F}} = \frac{2 + 1}{36.0 \mu \text{F}} \quad \text{or} \quad C_{\text{eq}} = 12.0 \mu \text{F} \]

When this series combination is connected to a 12.0-V battery, the total stored energy is

\[ \text{Total energy stored} = \frac{1}{2} C_{\text{eq}} (\Delta V)^2 = \frac{1}{2} (12.0 \times 10^{-6} \text{ F}) (12.0 \text{ V})^2 = 8.64 \times 10^{-5} \text{ J} \]

(b) The charge stored on each of the two capacitors in the series combination is

\[ Q_1 = Q_2 = Q_{\text{total}} = C_{\text{eq}} (\Delta V) = (12.0 \mu \text{F}) (12.0 \text{ V}) = 144 \mu \text{C} = 1.44 \times 10^{-4} \mu \text{C} \]

and the energy stored in each of the individual capacitors is

Energy stored in \( C_1 = \frac{Q_1^2}{2C_1} = \frac{(1.44 \times 10^{-4} \mu \text{C})^2}{2(18.0 \times 10^{-6} \mu \text{F})} = 5.76 \times 10^{-4} \text{ J} \]

and Energy stored in \( C_2 = \frac{Q_2^2}{2C_2} = \frac{(1.44 \times 10^{-4} \mu \text{C})^2}{2(36.0 \times 10^{-6} \mu \text{F})} = 2.88 \times 10^{-4} \text{ J} \]

Energy stored in \( C_1 \) + Energy stored in \( C_2 = 5.76 \times 10^{-4} \text{ J} + 2.88 \times 10^{-4} \text{ J} = 8.64 \times 10^{-4} \text{ J} \), which is the same as the total stored energy found in part (a). This must be true if the computed equivalent capacitance is truly equivalent to the original combination.

(c) If \( C_1 \) and \( C_2 \) had been connected in parallel rather than in series, the equivalent capacitance would have been \( C_{\text{eq}} = C_1 + C_2 = 18.0 \mu \text{F} + 36.0 \mu \text{F} = 54.0 \mu \text{F} \). If the total energy stored \( \frac{1}{2} C_{\text{eq}} (\Delta V)^2 \) in this parallel combination is to be the same as was stored in the original series combination, it is necessary that

\[ \Delta V = \sqrt{\frac{2 \text{Total energy stored}}{C_{\text{eq}}}} = \sqrt{\frac{2(8.64 \times 10^{-5} \mu \text{J})}{54.0 \times 10^{-6} \mu \text{F}}} = 5.66 \text{ V} \]

Since the two capacitors in parallel have the same potential difference across them, the energy stored in the individual capacitors \( \frac{1}{2} C (\Delta V)^2 \) is directly proportional to their capacitances. The larger capacitor, \( C_2 \), stores the most energy in this case.
16.47  
(a) The energy initially stored in the capacitor is

\[
(\text{Energy stored})_1 = \frac{Q^2}{2C_i} = \frac{1}{2} C_i (\Delta V)^2 = \frac{1}{2} (3.00 \ \mu\text{F})(6.00 \ \text{V})^2 = 54.0 \ \mu\text{J}
\]

(b) When the capacitor is disconnected from the battery, the stored charge becomes isolated with no way off the plates. Thus, the charge remains constant at the value \(Q\) as long as the capacitor remains disconnected. Since the capacitance of a parallel plate capacitor is

\[
C = \varepsilon_0 \frac{A}{d},
\]

when the distance \(d\) separating the plates is doubled, the capacitance is decreased by a factor of 2 (i.e., \(C_f = C_i/2 = 1.50 \ \mu\text{F}\)). The stored energy (with \(Q\) unchanged) becomes

\[
(\text{Energy stored})_2 = \frac{Q^2}{2C_f} = \frac{Q^2}{2(C_i/2)} = 2 (\text{Energy stored})_1 = 108 \ \mu\text{J}
\]

(c) When the capacitor is reconnected to the battery, the potential difference between the plates is reestablished at the original value of \(\Delta V = (\Delta V)_i = 6.00 \ \text{V}\), while the capacitance remains at \(C_f = C_i/2 = 1.50 \ \mu\text{F}\). The energy stored under these conditions is

\[
(\text{Energy stored})_3 = \frac{1}{2} C_f (\Delta V)^2 = \frac{1}{2} (1.50 \ \mu\text{F})(6.00 \ \text{V})^2 = 27.0 \ \mu\text{J}
\]

16.48  
The energy transferred to the water is

\[
W = \frac{1}{100} \left[ \frac{1}{2} Q (\Delta V) \right] = \frac{(50.0 \ \text{C})(1.00 \times 10^4 \ \text{V})}{200} = 2.50 \times 10^3 \ \text{J}
\]

Thus, if \(m\) is the mass of water boiled away,

\[
W = m[c(\Delta T) + L_v]
\]

becomes

\[
2.50 \times 10^3 \ \text{J} = m \left[ \frac{4186 \ \text{J}}{\text{kg} \cdot ^\circ\text{C}} \right] \left[ 100^\circ\text{C} - 30.0^\circ\text{C} \right] + 2.26 \times 10^6 \ \text{J/kg}
\]

giving

\[
m = \frac{2.50 \times 10^3 \ \text{J}}{2.55 \ \text{J/kg}} = 9.79 \ \text{kg}
\]

16.49  
(a) Note that the charge on the plates remains constant at the original value, \(Q_0\), as the dielectric is inserted. Thus, the change in the potential difference, \(\Delta V = Q/C\), is due to a change in capacitance alone. The ratio of the final and initial capacitances is

\[
\frac{C_f}{C_i} = \frac{\kappa \varepsilon_0 A/d}{\varepsilon_0 A/d} = \kappa \quad \text{and} \quad \frac{C_f}{C_i} = \frac{Q_f/(\Delta V)_f}{Q_i/(\Delta V)_i} = \frac{(\Delta V)_f}{(\Delta V)_i} = \frac{85.0 \ \text{V}}{25.0 \ \text{V}} = 3.40
\]

Thus, the dielectric constant of the inserted material is \(\kappa = 3.40\), and the material is probably [nylon](see Table 16.1).

(b) If the dielectric only partially filled the space between the plates, leaving the remaining space air-filled, the equivalent dielectric constant would be somewhere between \(\kappa = 1.00\) (air) and \(\kappa = 3.40\). The resulting potential difference would then lie somewhere between \((\Delta V)_i = 85.0 \ \text{V}\) and \((\Delta V)_f = 25.0 \ \text{V}\).
16.50 (a) The capacitance of the capacitor while air-filled is

\[ C_a = \frac{\varepsilon_0 A}{d} = \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right) \left(25.0 \times 10^{-4} \text{ m}^2\right) \left(1.50 \times 10^{-3} \text{ m}\right) = 1.48 \times 10^{-12} \text{ F} = 1.48 \text{ pF} \]

The original charge stored on the plates is

\[ Q_0 = C_a (\Delta V) = \left(1.48 \times 10^{-12} \text{ F}\right) \left(2.50 \times 10^3 \text{ V}\right) = 370 \times 10^{-12} \text{ C} = 370 \text{ pC} \]

Since distilled water is an insulator, introducing it between the isolated capacitor plates does not allow the charge to change. Thus, the final charge is \( Q_f = 370 \text{ pC} \).

(b) After immersion distilled water \((\kappa = 80\) — see Table 16.1), the new capacitance is

\[ C_f = \kappa C_a = (80) \left(1.48 \text{ pF}\right) = 118 \text{ pF} \]

and the new potential difference is \((\Delta V)_f = \frac{Q_f}{C_f} = \frac{370 \text{ pC}}{118 \text{ pF}} = 3.14 \text{ V}\).

(c) The energy stored in a capacitor is: Energy stored = \(Q^2 / 2C\). Thus, the change in the stored energy due to immersion in the distilled water is

\[ \Delta E = \frac{Q_f^2}{2C_f} - \frac{Q_0^2}{2C_0} = \left(\frac{Q_f}{C_f}\right)^2 - \left(\frac{Q_0}{C_0}\right)^2 = \left(\frac{370 \times 10^{-12} \text{ C}}{118 \times 10^{-12} \text{ F}}\right)^2 - \left(\frac{1}{1.48 \times 10^{-12} \text{ F}}\right) = -4.57 \times 10^{-8} \text{ J} = -45.7 \times 10^{-9} \text{ J} = -45.7 \text{ nJ} \]

16.51 (a) The dielectric constant for Teflon® is \(\kappa = 2.1\), so the capacitance is

\[ C = \frac{\kappa \varepsilon_0 A}{d} = \left(2.1\right) \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right) \left(175 \times 10^{-4} \text{ m}^2\right) \left(0.040 \times 10^{-3} \text{ m}\right) \]

\[ C = 8.13 \times 10^{-9} \text{ F} = 8.13 \text{ nF} \]

(b) For Teflon®, the dielectric strength is \(E_{\text{max}} = 60.0 \times 10^6 \text{ V/m}\), so the maximum voltage is

\[ V_{\text{max}} = E_{\text{max}} d = \left(60.0 \times 10^6 \text{ V/m}\right) \left(0.040 \times 10^{-3} \text{ m}\right) \]

\[ V_{\text{max}} = 2.40 \times 10^3 \text{ V} = 2.40 \text{ kV} \]

16.52 Before the capacitor is rolled, the capacitance of this parallel plate capacitor is

\[ C = \frac{\kappa \varepsilon_0 A}{d} = \frac{\kappa \varepsilon_0 (w \times L)}{d} \]

where \(A\) is the surface area of one side of a foil strip. Thus, the required length is

\[ L = \frac{C \cdot d \cdot \varepsilon_0 w}{\kappa \varepsilon_0} = \frac{(9.50 \times 10^{-8} \text{ F})(0.025 \times 10^{-3} \text{ m})}{(3.70) (8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.00 \times 10^{-2} \text{ m})} = 1.04 \text{ m} \]
16.53 (a) \[ V = \frac{m}{\rho} = \frac{1.00 \times 10^{-12} \text{ kg}}{1100 \text{ kg/m}^3} = 9.09 \times 10^{-16} \text{ m}^3 \]

Since \( V = \frac{4\pi r^3}{3} \), the radius is \( r = \left[ \frac{3V}{4\pi} \right]^{1/3} \), and the surface area is

\[ A = 4\pi r^2 = 4\pi \left[ \frac{3V^{2/3}}{4\pi} \right] = \frac{3(9.09 \times 10^{-16} \text{ m}^3)}{4\pi} = 4.54 \times 10^{-16} \text{ m}^2 \]

(b) \[ C = \frac{\kappa \varepsilon_0 A}{d} \]

\[ = \frac{(5.00)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(4.54 \times 10^{-16} \text{ m}^2)}{100 \times 10^{-9} \text{ m}} = 2.01 \times 10^{-13} \text{ F} \]

(c) \[ Q = C(\Delta V) = (2.01 \times 10^{-13} \text{ F})(100 \times 10^{-3} \text{ V}) = 2.01 \times 10^{-14} \text{ C} \]

and the number of electronic charges is

\[ n = \frac{Q}{e} = \frac{2.01 \times 10^{-14} \text{ C}}{1.60 \times 10^{-19} \text{ C}} = 1.26 \times 10^5 \]

16.54 Since the capacitors are in parallel, the equivalent capacitance is

\[ C_{eq} = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A_1}{d} + \frac{\varepsilon_0 A_2}{d} + \frac{\varepsilon_0 A_3}{d} = \frac{\varepsilon_0 (A_1 + A_2 + A_3)}{d} \]

or \[ C_{eq} = \frac{\varepsilon_0 A}{d} \text{ where } A = A_1 + A_2 + A_3 \]

16.55 Since the capacitors are in series, the equivalent capacitance is given by

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 A} + \frac{d_3}{\varepsilon_0 A} = \frac{d_1 + d_2 + d_3}{\varepsilon_0 A} \]

or \[ C_{eq} = \frac{\varepsilon_0 A}{d} \text{ where } d = d_1 + d_2 + d_3 \]

16.56 (a) Please refer to the solution of Problem 16.37, where the following results were obtained:

\[ C_{eq} = 3.33 \mu\text{F} \quad Q_3 = Q_6 = 180 \mu\text{C} \quad Q_7 = Q_4 = 120 \mu\text{C} \]

The total energy stored in the full circuit is then

\[ \text{(Energy stored)}_{\text{total}} = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2} (3.33 \times 10^{-6} \text{ F})(90.0 \text{ V})^2 \]

\[ = 1.35 \times 10^{-2} \text{ J} = 13.5 \times 10^{-3} \text{ J} = 0.0135 \text{ mJ} \]

continued on next page
(b) The energy stored in each individual capacitor is

For \(2.00 \mu F\):
\[
\text{Energy stored} = \frac{Q^2}{2C} = \frac{(120 \times 10^{-6} \text{ C})^2}{2(2.00 \times 10^{-6} \text{ F})} = 3.60 \times 10^{-3} \text{ J} = 3.60 \text{ mJ}
\]

For \(3.00 \mu F\):
\[
\text{Energy stored} = \frac{Q^2}{2C} = \frac{(180 \times 10^{-6} \text{ C})^2}{2(3.00 \times 10^{-6} \text{ F})} = 5.40 \times 10^{-3} \text{ J} = 5.40 \text{ mJ}
\]

For \(4.00 \mu F\):
\[
\text{Energy stored} = \frac{Q^2}{2C} = \frac{(120 \times 10^{-6} \text{ C})^2}{2(4.00 \times 10^{-6} \text{ F})} = 1.80 \times 10^{-3} \text{ J} = 1.80 \text{ mJ}
\]

For \(6.00 \mu F\):
\[
\text{Energy stored} = \frac{Q^2}{2C} = \frac{(180 \times 10^{-6} \text{ C})^2}{2(6.00 \times 10^{-6} \text{ F})} = 2.70 \times 10^{-3} \text{ J} = 2.70 \text{ mJ}
\]

(c) The total energy stored in the individual capacitors is

\[
\text{Energy stored total} = 3.60 + 5.40 + 1.80 + 2.70 \text{ mJ} = 13.5 \text{ mJ} = (\text{Energy stored})_{\text{total}}
\]

Thus, the sums of the energies stored in the individual capacitors equals the total energy stored by the system.

16.57

In the absence of a dielectric, the capacitance of the parallel plate capacitor is
\[
C_0 = \frac{\varepsilon_0 A}{d}
\]

With the dielectric inserted, it fills one-third of the gap between the plates as shown in sketch (a) at the right. We model this situation as consisting of a pair of capacitors, \(C_1\) and \(C_2\), connected in series as shown in sketch (b) at the right. In reality, the lower plate of \(C_1\) and the upper plate of \(C_2\) are one and the same, consisting of the lower surface of the dielectric shown in sketch (a). The capacitors in the model of sketch (b) are given by:
\[
C_1 = \frac{\kappa \varepsilon_0 A}{d/3} = \frac{3\kappa \varepsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{\varepsilon_0 A}{2d/3} = \frac{3 \varepsilon_0 A}{2d}
\]

and the equivalent capacitance of the series combination is
\[
\frac{1}{C_{eq}} = \frac{d}{3\kappa \varepsilon_0 A} + \frac{2d}{3 \varepsilon_0 A} = \left(\frac{1}{\kappa} + 2\right) \left(\frac{d}{3 \varepsilon_0 A}\right) = \left(\frac{2\kappa + 1}{\kappa}\right) \frac{d}{3 \varepsilon_0 A} = \left(\frac{2\kappa + 1}{3\kappa}\right) \frac{d}{\varepsilon_0 A} = \left(\frac{2\kappa + 1}{3\kappa}\right) \frac{1}{C_0}
\]

and
\[
C_{eq} = \left(\frac{3\kappa}{2\kappa + 1}\right) C_0
\]
16.58 For the parallel combination: 
\[ C_p = C_1 + C_2 \]
which gives 
\[ C_2 = C_p - C_1 \]  \[\text{[1]}\]

For the series combination:
\[ \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \]
or 
\[ \frac{1}{C_2} = \frac{1}{C_s} \frac{1}{C_1} \]

Thus, we have 
\[ C_2 = \frac{C_1 C_s}{C_1 - C_s} \]
and equating this to Equation [1] above gives 
\[ C_2 = \frac{C_1 C_s}{C_1 - C_s} \]
or 
\[ C_p C_1 C_s - C_1^2 + C_p C_1 C_s = C_s C_1 \]

We write this result as: 
\[ C_2 = \frac{1}{2} C_p \pm \sqrt{\frac{1}{4} C_p^2 - C_1 C_s} \]
and use the quadratic formula to obtain
\[ C_2 = \frac{1}{2} C_p \pm \sqrt{\frac{1}{4} C_p^2 - C_1 C_s} \]

Then, Equation [1] gives
\[ C_2 = \frac{1}{2} C_p \pm \sqrt{\frac{1}{4} C_p^2 - C_1 C_s} \]

16.59 The charge stored on the capacitor by the battery is
\[ Q = C(\Delta V)_1 = C(100 \text{ V}) \]

This is also the total charge stored in the parallel combination when this charged capacitor is connected in parallel with an uncharged \( 10.0 \mu F \) capacitor. Thus, if \( (\Delta V)_2 \) is the resulting voltage across the parallel combination, \( Q = C_p (\Delta V)_2 \) gives
\[ C(100 \text{ V}) = (C + 10.0 \mu F)(30.0 \text{ V}) \]
or 
\[ (70.0 \text{ V}) C = (30.0 \text{ V})(10.0 \mu F) \]

and 
\[ C = \left( \frac{30.0 \text{ V}}{70.0 \text{ V}} \right)(10.0 \mu F) = 4.29 \mu F \]

16.60 (a) The 1.0-\( \mu \)C is located 0.50 m from point \( P \), so its contribution to the potential at \( P \) is 
\[ V_1 = k_\varepsilon \frac{q_1}{r_1} = \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( 1.0 \times 10^{-6} \text{ C} \right) \left( 0.50 \text{ m} \right) = 1.8 \times 10^4 \text{ V} \]

(b) The potential at \( P \) due to the \(-2.0-\mu C\) charge located 0.50 m away is 
\[ V_2 = k_\varepsilon \frac{q_2}{r_2} = \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \left( -2.0 \times 10^{-6} \text{ C} \right) \left( 0.50 \text{ m} \right) = -3.6 \times 10^4 \text{ V} \]

(c) The total potential at point \( P \) is 
\[ V_P = V_1 + V_2 = (+1.8 - 3.6) \times 10^4 \text{ V} = -1.8 \times 10^4 \text{ V} \]

(d) The work required to move a charge \( q = 3.0 \mu \text{C} \) to point \( P \) from infinity is 
\[ W = q(\Delta V) = q(V_P - V_0) = \left( 3.0 \times 10^{-6} \text{ C} \right) \left( -1.8 \times 10^4 \text{ V} - 0 \right) = -5.4 \times 10^2 \text{ J} \]
16.61 The stages for the reduction of this circuit are shown below.

Thus, \( C_{eq} = 6.25 \mu F \)

16.62 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by \( V = \frac{kq}{r} \), where \( r \) is the distance from the center of that surface and \( q \) is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge \(+Q\) and the outer shell having radius \( b \) and charge \(-Q\), the total electric potential is given by

\[
V = V_{\text{due to inner shell}} + V_{\text{due to outer shell}} = k \frac{Q}{r} + k \frac{(-Q)}{b} = kQ \left( \frac{1}{r} - \frac{1}{b} \right)
\]

The potential difference between the two shells is therefore

\[
\Delta V = V_{b=a} - V_{a=b} = kQ \left( \frac{1}{a} - \frac{1}{b} \right) - kQ \left( \frac{1}{b} - \frac{1}{b} \right) = kQ \left( \frac{b-a}{ab} \right)
\]

The capacitance of this device is given by

\[
C = \frac{Q}{\Delta V} = \frac{ab}{k \left( b-a \right)}
\]

(b) When \( b \gg a \), then \( b-a = b \). Thus, in the limit as \( b \to \infty \), the capacitance found above becomes

\[
C \to \frac{ab}{k \left( b-a \right)} = \frac{a}{k} = \frac{4\pi \varepsilon_0 a}{k}
\]

16.63 The energy stored in a charged capacitor is \( W = \frac{1}{2} C (\Delta V)^2 \). Hence,

\[
\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \times 300 J}{30.0 \times 10^{-6} F}} = 4.47 \times 10^3 V = 4.47 kV
\]
16.64 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150 \mu C}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200 \mu C = 350 \mu C$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350 \mu C}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350 \mu C}{150 \mu C}\right) \frac{\Delta V}{\Delta V} = \frac{350}{150} = 2.33$$

16.65 The charges initially stored on the capacitors are

$$Q_1 = C_1 (\Delta V) = (6.0 \mu F)(250 \text{ V}) = 1.5 \times 10^3 \mu C$$

and

$$Q_2 = C_2 (\Delta V) = (2.0 \mu F)(250 \text{ V}) = 5.0 \times 10^2 \mu C$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive plate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5 \times 10^3 \mu C - 5.0 \times 10^2 \mu C = 1.0 \times 10^3 \mu C$$

The equivalent capacitance of the parallel combination is $C_{eq} = C_1 + C_2 = 8.0 \mu F$. Thus, the final potential difference across each of the capacitors is

$$\Delta V' = \frac{Q}{C_{eq}} = \frac{1.0 \times 10^3 \mu C}{8.0 \mu F} = 125 \text{ V}$$

and the final charge on each capacitor is

$$Q_1' = C_1 (\Delta V') = (6.0 \mu F)(125 \text{ V}) = 750 \mu C = 0.75 \text{ mC}$$

and

$$Q_2' = C_2 (\Delta V') = (2.0 \mu F)(125 \text{ V}) = 250 \mu C = 0.25 \text{ mC}$$

16.66 The energy required to melt the lead sample is

$$W = m\left[c_p (\Delta T) + L_f\right]$$

$$= \left(6.00 \times 10^{-5} \text{ kg}\right)\left[(128 \text{ J/kg} \cdot ^\circ \text{C})(327.3^\circ \text{C} - 20.0^\circ \text{C}) + 24.5 \times 10^3 \text{ J/kg}\right]$$

$$= 0.383 \text{ J}$$

The energy stored in a capacitor is $W = \frac{1}{2} C (\Delta V)^2$, so the required potential difference is

$$\Delta V = \sqrt{\frac{2W}{C}} = \sqrt{\frac{2(0.383 \text{ J})}{52.0 \times 10^{-6} \text{ F}}} = 121 \text{ V}$$
When excess charge resides on a spherical surface that is far removed from any other charge, this excess charge is uniformly distributed over the spherical surface, and the electric potential at the surface is the same as if all the excess charge were concentrated at the center of the spherical surface.

In the given situation, we have two charged spheres, initially isolated from each other, with charges and potentials of:

\[ Q_1 = +6.00 \mu C, \quad V_1 = kQ_1/R_1 \quad \text{where} \quad R_1 = 12.0 \text{ cm}, \quad Q_2 = -4.00 \mu C, \]

and \[ V_2 = kQ_2/R_2, \quad \text{with} \quad R_2 = 18.0 \text{ cm}. \]

When these spheres are then connected by a long conducting thread, the charges are redistributed (yielding charges of \( Q'_1 \) and \( Q'_2 \), respectively) until the two surfaces come to a common potential \( V'_1 = kQ'_1/R_1 = V'_2 = kQ'_2/R_2 \). When equilibrium is established, we have:

From conservation of charge: \[ Q'_1 + Q'_2 = Q_1 + Q_2 \quad \Rightarrow \quad Q'_1 + Q'_2 = +2.00 \mu C \quad [1] \]

From equal potentials: \[ \frac{kQ'_1}{R_1} = \frac{kQ'_2}{R_2} \quad \Rightarrow \quad Q'_1 = \left( \frac{R_2}{R_1} \right) Q'_2 \quad \text{or} \quad Q'_2 = 1.50 Q'_1 \quad [2] \]

Substituting Equation [2] into [1] gives: \[ Q'_1 = \frac{2+0.00 \mu C}{2.50} = 0.800 \mu C \]

Then, Equation [2] gives: \[ Q'_2 = 1.50(0.800 \mu C) = 1.20 \mu C \]

The electric field between the plates is directed downward with magnitude

\[ |E_x| = \frac{\Delta V}{d} = \frac{100 \text{ V}}{2.00 \times 10^{-3} \text{ m}} = 5.00 \times 10^4 \text{ N/m} \]

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

\[ a_y = \frac{F_y}{m_e} = \frac{qE_y}{m_e} = \frac{(-1.60 \times 10^{-19} \text{ C})(-5.00 \times 10^4 \text{ N/m})}{9.11 \times 10^{-31} \text{ kg}} = +8.78 \times 10^{15} \text{ m/s}^2 \]

(a) At the closest approach to the bottom plate, \( v_y = 0 \). Thus, the vertical displacement from point O is found from \( v_y^2 = v_{y0}^2 + 2a_y \Delta y \) as

\[ \Delta y = \frac{0 - (v_{y0} \sin \theta)^2}{2a_y} = \frac{-[(5.6 \times 10^6 \text{ m/s}) \sin 45^\circ]^2}{2(8.78 \times 10^{15} \text{ m/s}^2)} = -0.89 \text{ mm} \]

The minimum distance above the bottom plate is then

\[ d = \frac{D}{2} + \Delta y = 1.00 \text{ mm} - 0.89 \text{ mm} = 0.11 \text{ mm} \]

(b) The time for the electron to go from point O to the upper plate is found from \( \Delta y = v_{y0}t + \frac{1}{2}a_y t^2 \) as

\[ +1.00 \times 10^{-3} \text{ m} = \left[ -\left(5.6 \times 10^6 \frac{\text{m}}{\text{s}}\right) \sin 45^\circ \right] t + \frac{1}{2} \left(8.78 \times 10^{15} \frac{\text{m}}{\text{s}^2}\right) t^2 \]

Solving for \( t \) gives a positive solution of \( t = 1.11 \times 10^{-9} \text{ s} \). The horizontal displacement from point O at this time is

\[ \Delta x = v_{y0}t = \left[ (5.6 \times 10^6 \text{ m/s}) \cos 45^\circ \right](1.11 \times 10^{-9} \text{ s}) = 4.4 \text{ mm} \]