1. (30 points) Knowing that $\alpha = 50^\circ$ and that boom $AC$ exerts on pin $C$ a force directed along line $AC$, determine the force $P$ such that the tension in the cable is 300 lb.

Solution:

In the free-body diagram for the pin at $C$, there are three forces, the sum of which should vanish. The three forces form a triangle with two of the angles of the triangle identified as $100^\circ$ and $30^\circ$. The third is then $50^\circ$. The Law of Sines can now be used to write

$$\frac{P}{\sin 100^\circ} = \frac{300 \text{ lb}}{\sin 50^\circ} = \frac{F_{AC}}{\sin 30^\circ}$$

$$P = 386 \text{ lb}$$
2. (25 points, Partial Credit) To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at $B$ a 175-N force directed along line $AB$. Replace that force with an equivalent force-couple system at $C$.

Solution:

Since a stick cannot pull the doorknob, the 175 N force is in the direction from $A$ to $B$, and can be written as

$$\vec{F} = 175 \cdot \frac{73\hat{i} + 990\hat{j} - 494\hat{k}}{\sqrt{73^2 + 990^2 + 494^2}} = (11.5\hat{i} + 156\hat{j} - 78.0\hat{k})\text{N}$$

The displacement from $C$ to $B$ is

$$\vec{r}_{B/C} = (683\hat{i} - 860\hat{j} + 0\hat{k}) \text{ mm}$$

The moment about $C$ is

$$\vec{M}_C = \vec{r}_{B/C} \times \vec{F}$$

$$= (683\hat{i} - 860\hat{j}) \times (11.5\hat{i} + 156\hat{j} - 78.0\hat{k}) \text{ mm} \cdot \text{N}$$

$$= (67.1\hat{i} + 53.3\hat{j} + 116\hat{k}) \text{ m} \cdot \text{N}$$
3. (20 points, Partial Credit) Two 150-mm-diameter pulleys are mounted on line shaft $AD$. The belts at $B$ and $C$ lie in vertical planes parallel to the $yz$ plane. Replace the belt forces shown with an equivalent force-couple system at $A$.

Solution:

The two forces on belt at $C$ combine to a total force of

$$\vec{F}_C = 370 N \cdot (-\sin 10^\circ \hat{j} - \cos 10^\circ \hat{k})$$

$$= (-64.2 \hat{j} - 364 \hat{k}) \ N$$

and a moment about $C$ of

$$\vec{M}_C = 75 \cdot 60 \hat{i} \ mm \cdot N \ .$$

When transferred to point $A$, the moment of these two forces is

$$\vec{M}_{AC} = 4500 \hat{i} + 405 \cdot 370(\cos 10^\circ \hat{j} - \sin 10^\circ \hat{k}) \ mm \cdot N$$

$$= (4.50 \hat{i} + 147.6 \hat{j} - 26.0 \hat{k}) \ m \cdot N$$

The two forces on belt at $B$ combine to a total of

$$\vec{F}_B = [(-240 \cos 20^\circ - 145) \hat{j} + 240 \sin 20^\circ \hat{k}] N = (-370 \hat{j} + 82.1 \hat{k}) N$$

and a moment about $B$ of

$$\vec{M}_B = 75 \cdot 95 \hat{i} \ mm \cdot N = 7.12 \hat{i} \ m \cdot N \ .$$

When transferred to point $A$, the couple of these two forces is

$$\vec{M}_{AB} = (7.12 \hat{i} - 14.78 \hat{j} - 66.7 \hat{k}) \ m \cdot N$$

The total force at point $A$ is

$$\vec{F}_A = \vec{F}_B + \vec{F}_C = (-434 \hat{j} - 282 \hat{k}) N$$

and the total couple at $A$ is

$$\vec{M}_A = (4.50 \hat{i} + 147.6 \hat{j} - 26.0 \hat{k} + 7.12 \hat{i} - 14.78 \hat{j} - 66.7 \hat{k}) \ m \cdot N$$

$$= (11.62 \hat{i} + 132.8 \hat{j} - 92.7 \hat{k}) \ m \cdot N$$
4. **(25 points)** A 50-kg crate is attached to the trolley-beam system shown. Knowing that \( a = 1.2 \) m, determine (a) the tension in cable \( CD \), (b) the reaction at \( B \).

Solution:

Three forces acting on the massless steel beam sum to zero force and zero moment. The moment about \( B \) vanishes, from which we write

\[
M_B = 0 = -T \cdot 1.8 \sin 55° + 50 \times 9.8 \cdot 1.2 \ (m \cdot N)
\]

\[
T = 399 \text{ N}
\]

\[
0 = B_x + 399 \sin 55° \quad B_x = -327 \text{ N}
\]

\[
0 = B_y - 50 \times 9.8 + 399 \cos 55° \quad B_y = 261 \text{ N}
\]

Or, \( B = \sqrt{327^2 + 261^2} = 419 \text{ N} \) at an angle of \( 180° - \tan^{-1}(261/327) = 141.4° \).
5. (10 points, Extra Credit) A 450-lb load hangs from the corner \( C \) of a rigid piece of pipe \( ABCD \) which has been bent as shown. The pipe is supported by the ball-and-socket joints \( A \) and \( D \), which are fastened, respectively, to the floor and to a vertical wall and by a cable attached at the midpoint \( E \) of the portion \( BC \) of the pipe and at a point \( G \) on the wall. Determine (a) where \( G \) should be located if the tension in the cable is to be minimum, (b) the corresponding minimum value of the tension.

Solution:

The total moment about the line \( AD \), which has a direction unit vector of

\[
\mathbf{\lambda}_{AD} = \frac{12\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}}{\sqrt{12^2 + 12^2 + 6^2}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k},
\]

should vanish.

The moment due to the 450 lb force is

\[
\mathbf{\lambda}_{AD} \cdot \mathbf{\bar{r}}_{C/A} \times (-450\mathbf{j}) = \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \times (12\mathbf{i} + 12\mathbf{j} - (-450\mathbf{j})) \text{ ft} \cdot \text{lb} = 1800 \text{ ft} \cdot \text{lb},
\]

which should be balanced by the moment due to tension. Maximum moment is applied at point \( E \), if the direction of the force (i.e. \( EG \)) is made parallel to

\[
(6\mathbf{i} + 12\mathbf{j}) \times \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) = -4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} = 6 \text{ ft} \left\{-\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right\}
\]

Since 6 ft is the lever arm, the minimum value of tension is

\[
1800 \text{ ft} \cdot \text{lb} / 6 \text{ ft} = 300 \text{ lb}
\]

To find the location \( G \), we notice that the z-component of \( \mathbf{\bar{r}}_{G/E} \) is -6 ft, which means that

\[
\mathbf{\bar{r}}_{G/E} = \left(-6.00\mathbf{i} + 3.00\mathbf{j} - 6.00\mathbf{k}\right) \text{ ft}
\]

or

\[
\mathbf{\bar{r}}_{G/D} = \left(-12.0\mathbf{i} + 3.00\mathbf{j}\right) \text{ ft}
\]