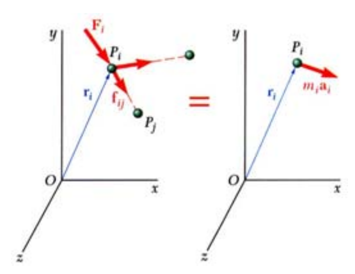
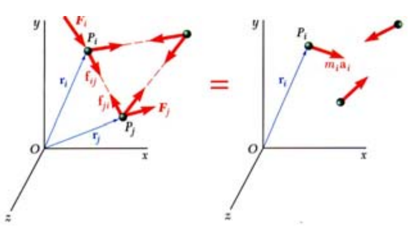
Chapter 14, Systems of Particles

- The *effective force* of a particle is defined as the product of it mass and acceleration. It will be shown that the *system of external forces* acting on a system of particles is *equipollent* with the *system of effective forces* of the system.
- The *mass center* of a system of particles will be defined and its motion described.
- Application of the *work-energy principle* and the *impulse-momentum principle* to a system of particles will be described. Result obtained are also applicable to a system of rigidly connected particles, i.e., a *rigid body*.
- Analysis methods will be presented for *variable systems of particles*, i.e., systems in which the particles included in the system change.

Application of Newton's Laws. Effective Forces





- Newton's second law for each particle P_i in a system of n particles,

$$\vec{F}_i + \sum_{j=1}^{n} \vec{f}_{ij} = m_i \vec{a}_i$$

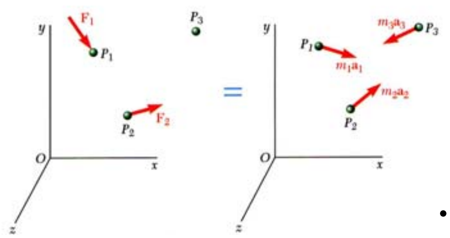
$$\vec{r}_i \times \vec{F}_i + \sum_{j=1}^{n} \left(\vec{r}_i \times \vec{f}_{ij} \right) = \vec{r}_i \times m_i \vec{a}_i$$

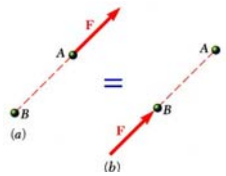
$\vec{F}_i =$ external force $\qquad \vec{f}_{ij} =$ internal forces

$m_i \vec{a}_i =$ effective force

- The system of external and internal forces on a particle is *equivalent* to the effective force of the particle.
- The system of external and internal forces acting on the entire system of particles is *equivalent* to the system of effective forces.

Application of Newton's Laws. Effective Forces





- Summing over all the elements,

$$\sum_{i=1}^{n}\vec{F}_i + \sum_{i=1}^{n}\sum_{j=1}^{n}\vec{f}_{ij} = \sum_{i=1}^{n} m_i\vec{a}_i$$

$$\sum_{i=1}^{n}\left(\vec{r}_i \times \vec{F}_i\right) + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\vec{r}_i \times \vec{f}_{ij}\right) = \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{a}_i\right)$$

- Since the internal forces occur in equal and opposite collinear pairs, the resultant force and couple due to the internal forces are zero,

$$\sum \vec{F}_i = \sum m_i\vec{a}_i$$
$$\sum\left(\vec{r}_i \times \vec{F}_i\right) = \sum\left(\vec{r}_i \times m_i\vec{a}_i\right)$$

- The system of external forces and the system of effective forces are *equipollent* by not *equivalent*.

Linear & Angular Momentum

- Linear momentum of the system of particles,

$$\vec{L} = \sum_{i=1}^{n} m_i\vec{v}_i$$

$$\dot{\vec{L}} = \sum_{i=1}^{n} m_i\dot{\vec{v}}_i = \sum_{i=1}^{n} m_i\vec{a}_i$$

- Resultant of the external forces is equal to rate of change of linear momentum of the system of particles,

$$\sum \vec{F} = \dot{\vec{L}}$$

- Angular momentum about fixed point *O* of system of particles,

$$\vec{H}_O = \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{v}_i\right)$$

$$\dot{\vec{H}}_O = \sum_{i=1}^{n}\left(\dot{\vec{r}}_i \times m_i\vec{v}_i\right) + \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\dot{\vec{v}}_i\right)$$

$$= \sum_{i=1}^{n}\left(\vec{r}_i \times m_i\vec{a}_i\right)$$

- Moment resultant about fixed point *O* of the external forces is equal to the rate of change of angular momentum of the system of particles,

$$\sum \vec{M}_O = \dot{\vec{H}}_O$$

Motion of the Mass Center of a System of Particles

- Mass center G of system of particles is defined by position vector $\vec{r}_G$ which satisfies

$$m\vec{r}_G = \sum_{i=1}^{n} m_i \vec{r}_i$$

- Differentiating twice,

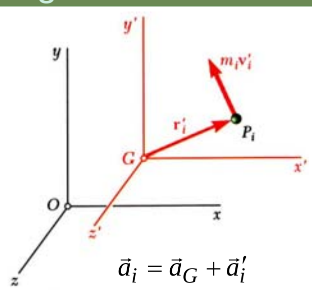
$$m\dot{\vec{r}}_G = \sum_{i=1}^{n} m_i \dot{\vec{r}}_i$$

$$m\vec{v}_G = \sum_{i=1}^{n} m_i \vec{v}_i = \vec{L}$$

$$m\vec{a}_G = \dot{\vec{L}} = \sum \vec{F}$$

- The mass center moves as if the entire mass and all of the external forces were concentrated at that point.

Angular Momentum About the Mass Center



$$\vec{a}_i = \vec{a}_G + \vec{a}_i'$$

- Consider the centroidal frame of reference *Gx'y'z'*, which translates with respect to the Newtonian frame *Oxyz*.
- The centroidal frame is not, in general, a Newtonian frame.

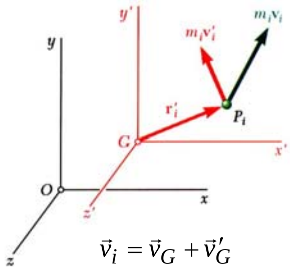
- The angular momentum of the system of particles about the mass center,

$$\vec{H}_G' = \sum_{i=1}^{n}\left(\vec{r}_i' \times m_i \vec{v}_i'\right)$$

$$\begin{aligned}\dot{\vec{H}}_G' &= \sum_{i=1}^{n}\left(\vec{r}_i' \times m_i \vec{a}_i'\right) = \sum_{i=1}^{n}\left(\vec{r}_i' \times m_i\left(\vec{a}_i - \vec{a}_G\right)\right) \\ &= \sum_{i=1}^{n}\left(\vec{r}_i' \times m_i \vec{a}_i\right) - \left(\sum_{i=1}^{n} m_i \vec{r}\,'\right) \times \vec{a}_G \\ &= \sum_{i=1}^{n}\left(\vec{r}_i' \times m_i \vec{a}_i\right) = \sum_{i=1}^{n}\left(\vec{r}_i' \times \vec{F}_i\right) \\ &= \sum \vec{M}_G\end{aligned}$$

- The moment resultant about G of the external forces is equal to the rate of change of angular momentum about G of the system of particles.

Angular Momentum About the Mass Center



$$\vec{v}_i = \vec{v}_G + \vec{v}_G'$$

- Angular momentum about *G* of the particles in their motion relative to the centroidal *Gx'y'z'* frame of reference,

$$\vec{H}_G' = \sum_{i=1}^{n} \left(\vec{r}_i' \times m_i \vec{v}_i' \right)$$

- Angular momentum about *G* of particles in their absolute motion relative to the Newtonian *Oxyz* frame of reference.

$$\begin{aligned} \vec{H}_G &= \sum_{i=1}^{n} \left(\vec{r}_i' \times m_i \vec{v}_i \right) \\ &= \sum_{i=1}^{n} \left(\vec{r}_i' \times m_i \left(\vec{v}_G + \vec{v}_i' \right) \right) \\ &= \left(\sum_{i=1}^{n} m_i \vec{r}_i' \right) \times \vec{v}_G + \sum_{i=1}^{n} \left(\vec{r}_i' \times m_i \vec{v}_i \right) \end{aligned}$$

$$\vec{H}_G = \vec{H}_G' = \sum \vec{M}_G$$

- Angular momentum about *G* of the particle momenta can be calculated with respect to either the Newtonian or centroidal frames of reference.

Conservation of Momentum

- If no external forces act on the particles of a system, then the linear momentum and angular momentum about the fixed point *O* are conserved.

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} = \text{constant} \qquad \vec{H}_O = \text{constant}$$

- In some applications, such as problems involving central forces,

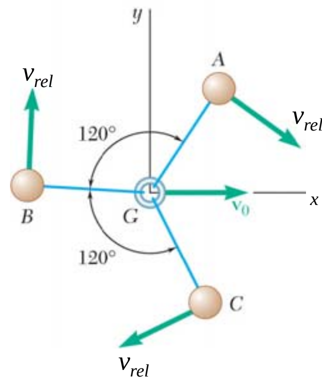
$$\dot{\vec{L}} = \sum \vec{F} \neq 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} \neq \text{constant} \qquad \vec{H}_O = \text{constant}$$

- Concept of conservation of momentum also applies to the analysis of the mass center motion,

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_G = \sum \vec{M}_G = 0$$
$$\vec{L} = m\vec{v}_G = \text{constant}$$
$$\vec{v}_G = \text{constant} \qquad \vec{H}_G = \text{constant}$$

Concept Question

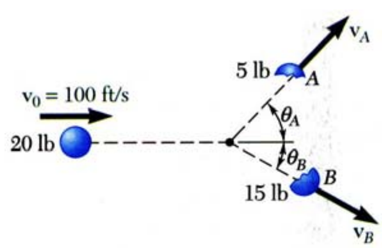
Three small identical spheres *A*, *B*, and *C*, which can slide on a horizontal, frictionless surface, are attached to three 200-mm-long strings, which are tied to a ring *G*. Initially, each of the spheres rotate clockwise about the ring with a relative velocity of v_{rel}.

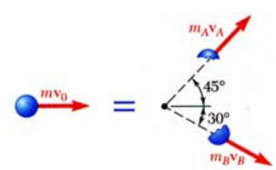


Which of the following is true?

a) $\boxed{\text{The linear momentum of the system is in the positive x direction}}$
b) The angular momentum of the system is in the positive y direction
c) The angular momentum of the system about G is zero
d) The linear momentum of the system is zero

Sample Problem 14.2





$$m_A\vec{v}_A + m_B\vec{v}_B = m\vec{v}_0$$
$$(5/g)\vec{v}_A + (15/g)\vec{v}_B = (20/g)\vec{v}_0$$

A 20-lb projectile is moving with a velocity of 100 ft/s when it explodes into 5 and 15-lb fragments. Immediately after the explosion, the fragments travel in the directions θ_A = 45° and θ_B = 30°.

Determine the velocity of each fragment.

x components:

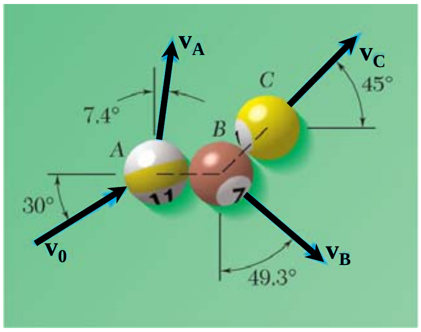
$$5v_A \cos 45° + 15v_B \cos 30° = 20(100)$$

y components:

$$5v_A \sin 45° - 15v_B \sin 30° = 0$$

- Solve the equations simultaneously for th fragment velocities.

Problem



In a game of pool, ball *A* is moving with a velocity $\mathbf{v}_0$ when it strikes balls *B* and *C*, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and that v_0 = 12 ft/s and v_C= 6.29 ft/s, determine the magnitude of the velocity of (*a*) ball *A*, (*b*) ball *B*.

$$m(12 \text{ ft/s})\cos 30° = mv_A \sin 7.4° + mv_B \sin 49.3° + m(6.29)\cos 45°$$

$$0.12880v_A + 0.75813v_B = 5.9446$$

$$m(12 \text{ ft/s})\sin 30° = mv_A \cos 7.4° - mv_B \cos 49.3° + m(6.29)\sin 45°$$

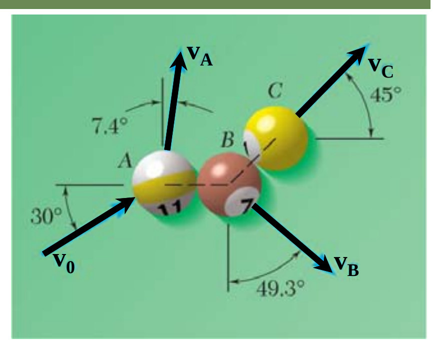
$$0.99167v_A - 0.65210v_B = 1.5523$$

$$v_A = 6.05 \text{ ft/s}$$

Concept Question

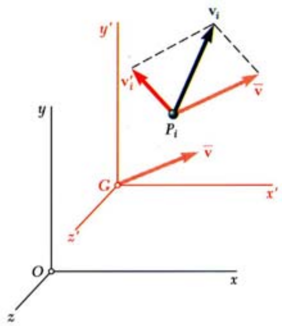
In a game of pool, ball *A* is moving with a velocity $\mathbf{v}_0$ when it strikes balls *B* and *C*, which are at rest and aligned as shown.

After the impact, what is true about the overall center of mass of the system of three balls?



a) The overall system CG will move in the same direction as v_0
b) The overall system CG will stay at a single, constant point
c) There is not enough information to determine the CG location

Kinetic Energy



$$\vec{v}_i = \vec{v}_G + \vec{v}_i'$$

- Kinetic energy of a system of particles,

$$T = \tfrac{1}{2}\sum_{i=1}^{n} m_i\left(\vec{v}_i \bullet \vec{v}_i\right) = \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i^2$$

- Expressing the velocity in terms of the centroidal reference frame,

$$\begin{aligned} T &= \tfrac{1}{2}\sum_{i=1}^{n}\left[m_i\left(\vec{v}_G + \vec{v}_i'\right)\bullet\left(\vec{v}_G + \vec{v}_i'\right)\right] \\ &= \tfrac{1}{2}\left(\sum_{i=1}^{n} m_i\right)v_G^2 + \vec{v}_G \bullet \sum_{i=1}^{n} m_i\vec{v}_i' + \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i'^2 \\ &= \tfrac{1}{2} m\vec{v}_G^2 + \tfrac{1}{2}\sum_{i=1}^{n} m_i v_i'^2 \end{aligned}$$

- Kinetic energy is equal to kinetic energy of mass center plus kinetic energy relative to the centroidal frame.

Work-Energy Principle. Conservation of Energy

- Principle of work and energy can be applied to each particle P_i,

  $$T_1 + U_{1\to 2} = T_2$$

  where $U_{1\to 2}$ represents the work done by the internal forces $\vec{f}_{ij}$ and the resultant external force $\vec{F}_i$ acting on P_i.

- Principle of work and energy can be applied to the entire system by adding the kinetic energies of all particles and considering the work done by all external and internal forces.

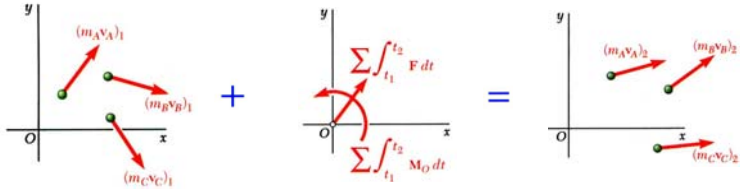
- Although $\vec{f}_{ij}$ and $\vec{f}_{ji}$ are equal and opposite, the work of these forces will not, in general, cancel out.

- If the forces acting on the particles are conservative, the work is equal to the change in potential energy and

  $$T_1 + V_1 = T_2 + V_2$$

  which expresses the principle of conservation of energy for the system of particles.

Principle of Impulse and Momentum



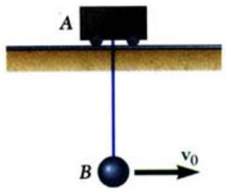
$$\sum \vec{F} = \dot{\vec{L}} \qquad \sum \vec{M}_O = \dot{\vec{H}}_O$$

$$\sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2 - \vec{L}_1 \qquad \sum \int_{t_1}^{t_2} \vec{M}_O dt = \vec{H}_2 - \vec{H}_1$$

$$\vec{L}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2 \qquad \vec{H}_1 + \sum \int_{t_1}^{t_2} \vec{M}_O dt = \vec{H}_2$$

- The momenta of the particles at time t_1 and the impulse of the forces from t_1 to t_2 form a system of vectors *equipollent* to the system of momenta of the particles at time t_2 .

Sample Problem 14.4



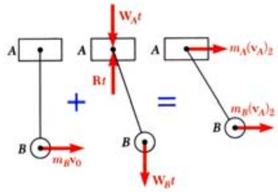
Ball *B*, of mass m_B,is suspended from a cord, of length *l*, attached to cart *A*, of mass m_A, which can roll freely on a frictionless horizontal tract. While the cart is at rest, the ball is given an initial velocity $v_0 = \sqrt{2gl}$.

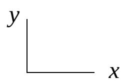
Determine *(a)* the velocity of *B* as it reaches it maximum elevation, and *(b)* the maximum vertical distance *h* through which *B* will rise.

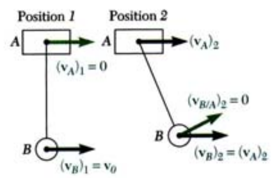
SOLUTION:

- With no external horizontal forces, it follows from the impulse-momentum principle that the horizontal component of momentum is conserved. This relation can be solved for the velocity of *B* at its maximum elevation.
- The conservation of energy principle can be applied to relate the initial kinetic energy to the maximum potential energy. The maximum vertical distance is determined from this relation.

Sample Problem 14.4







SOLUTION:

- With no external horizontal forces, it follows from the impulse-momentum principle that the horizontal component of momentum is conserved. This relation can be solved for the velocity of *B* at its maximum elevation.

$$\vec{L}_1 + \sum \int_{t_1}^{t_2} \vec{F} dt = \vec{L}_2$$

x component equation:

$$m_A v_{A,1} + m_B v_{B,1} = m_A v_{A,2} + m_B v_{B,2}$$

Velocities at positions 1 and 2 are

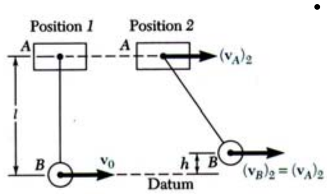
$$v_{A,1} = 0 \quad v_{B,1} = v_0$$

$$v_{B,2} = v_{A,2} + v_{B/A,2} = v_{A,2}$$ (velocity of *B* relative to *A* is zero at position 2)

$$m_B v_0 = (m_A + m_B) v_{A,2}$$

$$\boxed{v_{A,2} = v_{B,2} = \frac{m_B}{m_A + m_B} v_0}$$

Sample Problem 14.4



- The conservation of energy principle can be applied to relate the initial kinetic energy to the maximum potential energy.

$$T_1 + V_1 = T_2 + V_2$$

Position 1 -	Potential Energy:	$V_1 = m_A g l$
	Kinetic Energy:	$T_1 = \frac{1}{2} m_B v_0^2$
Position 2 -	Potential Energy:	$V_2 = m_A g l + m_B g h$
	Kinetic Energy:	$T_2 = \frac{1}{2}(m_A + m_B) v_{A,2}^2$

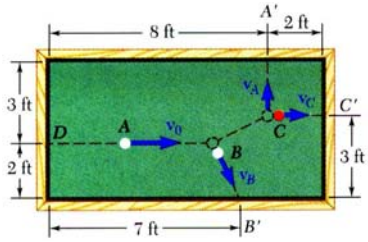
$$\frac{1}{2} m_B v_0^2 + m_A g l = \frac{1}{2}(m_A + m_B) v_{A,2}^2 + m_A g l + m_B g h$$

$$h = \frac{v_0^2}{2g} - \frac{m_A + m_B}{m_B} \frac{v_{A,2}^2}{2g} = \frac{v_0^2}{2g} - \frac{m_A + m_B}{2g\, m_B} \left(\frac{m_B}{m_A + m_B} v_0 \right)^2$$

$$h = \frac{v_0^2}{2g} - \frac{m_B}{m_A + m_B} \frac{v_0^2}{2g}$$

$$\boxed{h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g}}$$

Sample Problem 14.5



Ball A has initial velocity v_0 = 10 ft/s parallel to the axis of the table. It hits ball B and then ball C which are both at rest. Balls A and C hit the sides of the table squarely at A' and C' and ball B hits obliquely at B'.

Assuming perfectly elastic collisions, determine velocities v_A, v_B, and v_C with which the balls hit the sides of the table.

<u>SOLUTION</u>:

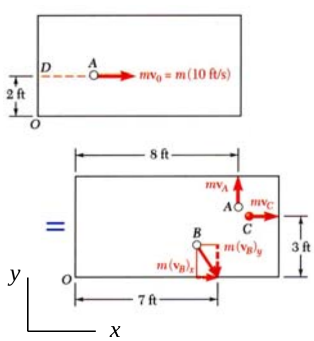
- There are four unknowns: v_A, $v_{B,x}$, $v_{B,y}$, and v_C.
- Solution requires four equations: conservation principles for linear momentum (two component equations), angular momentum, and energy.
- Write the conservation equations in terms of the unknown velocities and solve simultaneously.

Sample Problem 14.5

<u>SOLUTION</u>:

- There are four unknowns: v_A, $v_{B,x}$, $v_{B,y}$, and v_C.

$$\vec{v}_A = v_A \vec{j}$$
$$\vec{v}_B = v_{B,x}\vec{i} + v_{B,y}\vec{j}$$
$$\vec{v}_C = v_C \vec{i}$$



- The conservation of momentum and energy equations,

$$\vec{L}_1 + \sum \int \vec{F} dt = \vec{L}_2$$
$$mv_0 = mv_{B,x} + mv_C \qquad 0 = mv_A - mv_{B,y}$$

$$\vec{H}_{O,1} + \sum \int \vec{M}_O dt = \vec{H}_{O,2}$$
$$-(2\,\text{ft})mv_0 = (8\,\text{ft})mv_A - (7\,\text{ft})mv_{B,y} - (3\,\text{ft})mv_C$$

$$T_1 + V_1 = T_2 + V_2$$
$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_A^2 + \tfrac{1}{2}m\left(v_{B,x}^2 + v_{B,y}^2\right) + \tfrac{1}{2}mv_C^2$$

Solving the first three equations in terms of v_C,

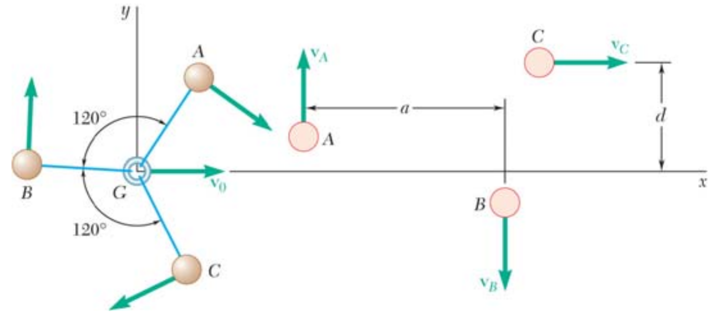
$$v_A = v_{B,y} = 3v_C - 20 \qquad v_{B,x} = 10 - v_C$$

Substituting into the energy equation,

$$2(3v_C - 20)^2 + (10 - v_C)^2 + v_C^2 = 100$$
$$20v_C^2 - 260v_C + 800 = 0$$

$$\boxed{\begin{array}{l} v_A = 4\,\text{ft/s} \quad v_C = 8\,\text{ft/s} \\ \vec{v}_B = (2\vec{i} - 4\vec{j})\,\text{ft/s} \quad v_B = 4.47\,\text{ft/s} \end{array}}$$

Problem

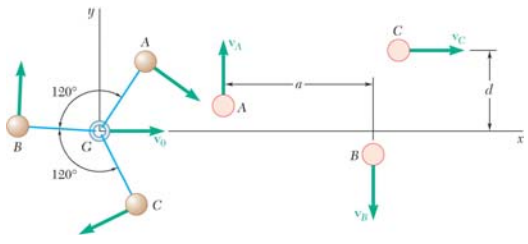


Three small identical spheres *A*, *B*, and *C*, which can slide on a horizontal, frictionless surface, are attached to three 200-mm-long strings, which are tied to a ring *G*. Initially, the spheres rotate clockwise about the ring with a relative velocity of 0.8 m/s and the ring moves along the *x*-axis with a velocity $\mathbf{v_0}$= (0.4 m/s)**i.** Suddenly, the ring breaks and the three spheres move freely in the *xy* plane with *A* and *B* following paths parallel to the *y*-axis at a distance a= 346 mm from each other and *C* following a path parallel to the *x* axis. Determine (*a*) the velocity of each sphere, (*b*) the distance *d*.

Group Problem Solving

Given: $v_{Arel} = v_{Brel} = v_{Crel} = 0.8$ m/s, $\mathbf{v_0}$= (0.4 m/s)**i** , L= 200 mm, a= 346 mm

Find: v_A, v_B, v_C (after ring breaks), *d*



SOLUTION:

- There are four unknowns: v_A, v_B, v_B, d.
- Solution requires four equations: conservation principles for linear momentum (two component equations), angular momentum, and energy.
- Write the conservation equations in terms of the unknown velocities and solve simultaneously.

Apply the conservation of linear momentum equation – find L_0 before ring breaks

$$\mathbf{L}_0 = (3\text{m})\overline{\mathbf{v}} = 3m(0.4\mathbf{i}) = \text{m}(1.2\ \text{m/s})\mathbf{i}$$

What is L_f (after ring breaks)?

$$\mathbf{L}_f = mv_A\mathbf{j} - mv_B\mathbf{j} + mv_C\mathbf{i}$$

Problem Solving

Set $L_0 = L_f$

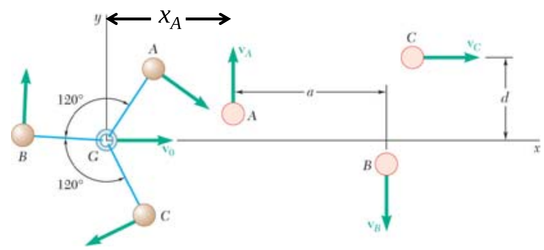
$$m(1.2 \text{ m/s})\mathbf{i} = mv_C\mathbf{i} + m(v_A - v_B)\mathbf{j}$$

From the y components,

$$v_A = v_B$$

From the x components,

$$v_C = 1.200 \text{ m/s} \qquad \mathbf{v}_C = 1.200 \text{ m/s}$$



Apply the conservation of angular momentum equation

H_0: $\quad (H_G)_0 = 3mlv_{rel} = 3m(0.2\,\text{m})(0.8 \text{ m/s}) = 0.480m$

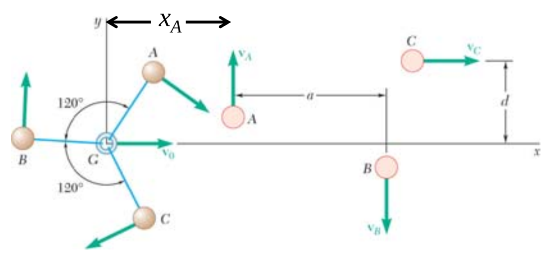
H_f: $\quad (H_G)_f = -mv_A\, x_A + mv_B(x_A + a) + mv_C d =$

Since $v_A = v_B$, *and* $v_C = 1.2$ m/s, then:

$$0.480m = 0.346mv_A + mv_C d$$
$$0.480 = 0.346v_A + 1.200d$$
$$d = 0.400 - 0.28833v_A$$

Problem Solving

Need another equation- try work-energy, where $T_0 = T_f$



T_0:

$$T_0 = \frac{1}{2}(3\text{m})\bar{v}^2 + 3\left(\frac{1}{2}mv_{rel}{}^2\right)$$
$$= \frac{3}{2}\text{m}\left(v_0^2 + v_{rel}{}^2\right) = \frac{3}{2}[(0.4)^2 + (0.8)^2]m = 1.200m$$

T_f:

$$T_f = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

Substitute in known values:

$$\frac{1}{2}\left[v_A^2 + v_A^2 + (1.200)^2\right] = 1.200$$
$$v_A^2 = 0.480$$
$$v_A = v_B = 0.69282 \text{ m/s}$$

Solve for d:

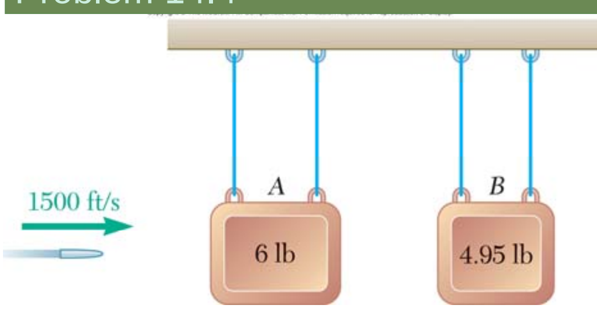
$$d = 0.400 - 0.28833(0.69282) = 0.20024 \text{ m}$$

$\mathbf{v}_A = 0.693$ m/s ↑	$\mathbf{v}_C = 1.200$ m/s ⟶
$\mathbf{v}_B = 0.693$ m/s ↓	$d = 0.200$ m

Variable Systems of Particles

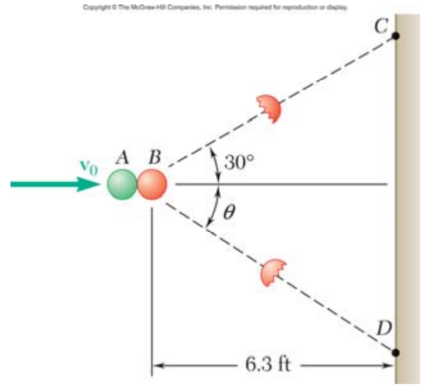
- Kinetics principles established so far were derived for constant systems of particles, i.e., systems which neither gain nor lose particles.
- A large number of engineering applications require the consideration of variable systems of particles, e.g., hydraulic turbine, rocket engine, etc.
- For analyses, consider auxiliary systems which consist of the particles instantaneously within the system plus the particles that enter or leave the system during a short time interval. The auxiliary systems, thus defined, are constant systems of particles.

Problem 14.4



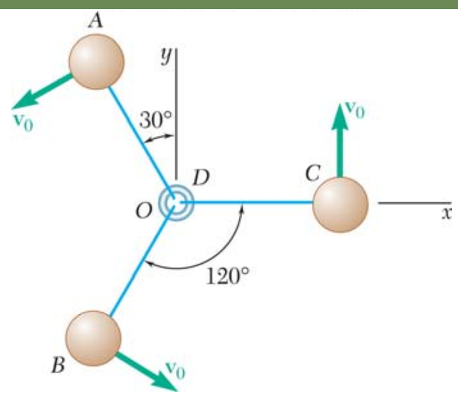
A bullet is fired with a horizontal velocity of 1500 ft/s through a 6-lb block *A* and becomes embedded in a 4.95-lb block *B*. Knowing that blocks *A* and *B* start moving with velocities of 5 ft/s and 9 ft/s, respectively, determine (a) the weight of the bullet, (b) its velocity as it travels from block A to block B.

Problem 14.22



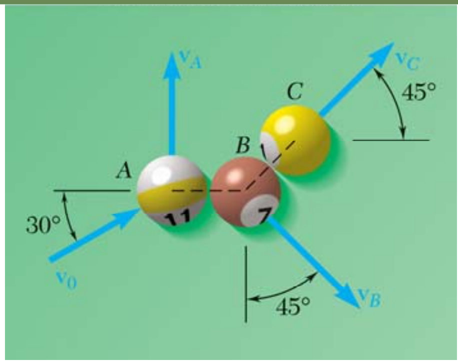
Two spheres, each of mass *m*, can slide freely on a frictionless horizontal surface. Sphere *A* is moving at a speed v_0 = 16 ft/s when it strikes sphere *B* which is at rest and the impact causes sphere B to break into two pieces, each of mass m/2. Knowing that 0.7 s after the collision one piece reaches point *C* and 0.9 s after the collision the other piece reaches point *D*, determine (a) the velocity of sphere *A* after the collision, (b) the angle θ and the speed of the two pieces after the collision.

Problem 14.50



Three small spheres *A*, *B*, and *C*, each of mass *m*, are connected to a small ring *D* of negligible mass by means of three inextensible inelastic cods of length *l*. The spheres can slide freely on a frictionless horizontal surface and are rotating initially at a speed v_0 about ring *D* which is at rest. Suddenly the cord *CD* breaks. After the other two cords have again become taut, determine (a) the speed of ring *D*, (b) the relative speed at which spheres *A* and *B* rotate about *D*, (c) the fraction of the original energy of spheres *A* and *B* which is dissipated when cords *AD* and

Problem 14.42



In a game of pool, ball *A* is moving with a velocity v_0 of magnitude 10 ft/s when it strikes balls *B* and *C* which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (i.e. conservation of energy), determine the magnitudes of the velocities v_A, v_B, and v_C.